DRONACHARYA
College of Engineering

Fluid Mechanics

Fundamental Concepts

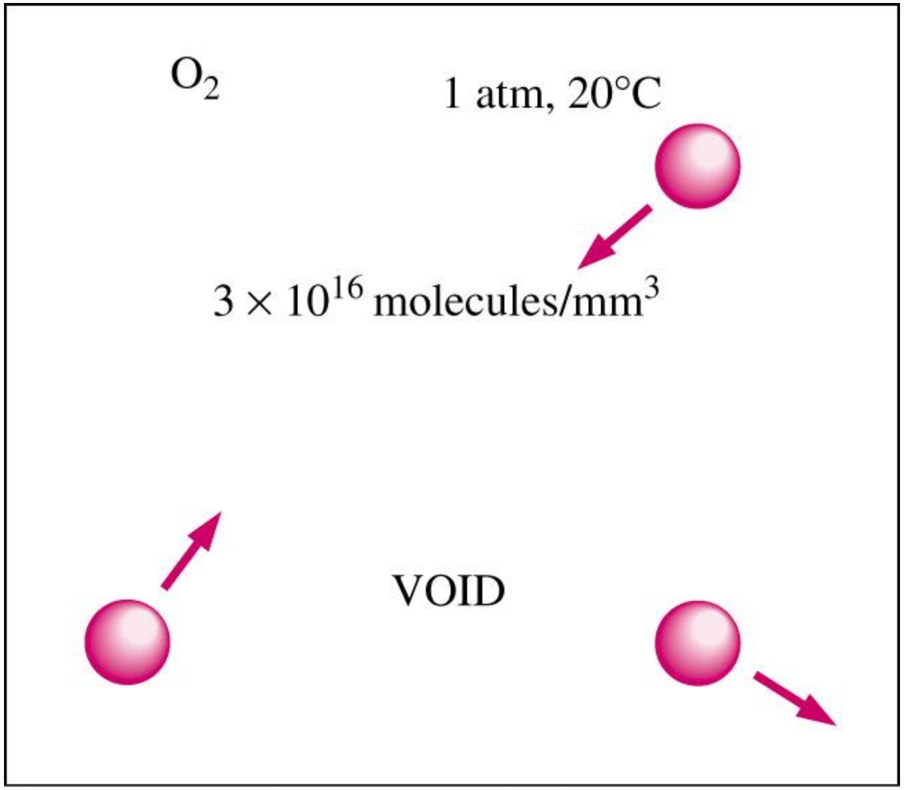
Property of the System

- ✓ Any characteristic of a system is called a **property.**
  - Familiar: pressure P, temperature T, volume V, and mass m.
  - Less familiar: viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, vapor pressure, surface tension.
- ✓ ***Intensive* properties** are independent of the mass of the system. Examples: temperature, pressure, and density.
- ✓ ***Extensive* properties** are those whose value depends on the size of the system.

  Examples: Total mass, total volume, and total momentum.
- ✓ Extensive properties per unit mass are called **specific properties**.

  Examples include specific volume $v = V/m$ and specific total energy $e=E/m$.

Fluid as a continuum or continuum based approach

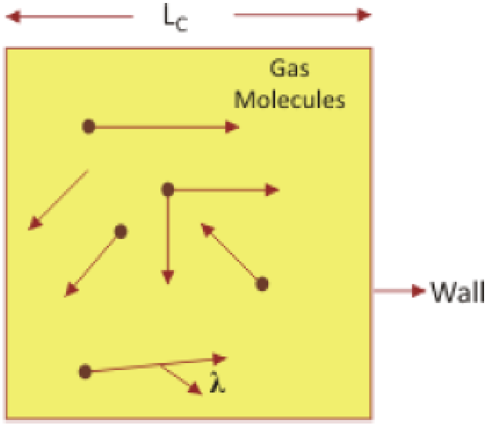


- ✓ Atoms are widely spaced in the gas phase.
- ✓ However, we can disregard the atomic nature of a substance.
- ✓ View it as a continuous, homogeneous matter with no holes, that is, a **continuum**.
- ✓ This allows us to treat properties as smoothly varying quantities.
- ✓ Continuum is valid as long as size of the system is large in comparison to distance between molecules.

✓ **Fluid** is **made of molecules**. However, for most of the engineering applications, when we speak of fluid's properties such as **density, or conditions such as pressure and temperature**, we **do not imply such properties or conditions of individual molecules**, but those of “fluid” as a whole.

✓ Refer to the average **or macroscopic aggregate effects** of the fluid molecules, reflected **in pressure, temperature, density, etc**.

✓ This approach is called **continuum based approach or fluid is treated as continuum**.



However, there is a restriction!!!!

✓ The continuum approach can be applied only when the **mean free path of the fluid** (largely, gas) **is smaller** (actually much smaller!!) **than the physical characteristic length of the system under consideration** (diameter of the tube in which the gas flows, or size of a container in which gas is stored)

✓ The continuum approach is usually not valid when **the gas pressure is very small (few millitorr like in a vacuum), or the aperture size is small** (like in an orifice)

✓ Mathematically, for the continuum approach based model to hold good, where is the mean free path of the gas molecule and L_C is the characteristic length of the system.

✓ Alternatively, Knudsen defined as: $\frac{\lambda}{L_C} \ll \mathbf{1}$

Density and Specific Gravity

- ✓ Density is defined as the ***mass per unit volume*** $\rho = m/V$ Density has units of kg/m^3

- ✓ Specific volume is defined as $\boldsymbol{v = 1/\rho = V/m}$.

- ✓ For a gas, density depends on temperature and pressure.

- ✓ **Specific gravity**, or relative density is defined as ***the ratio of the density of a substance to the density of some standard substance at a specified temperature*** (usually water at 4°C), i.e., $SG = \rho/\rho_{H_20}$. SG is a dimensionless quantity.

- ✓ The **specific weight** is defined as the weight per unit volume, i.e., $g_s = \rho g$ where g is the gravitational acceleration. g_s has units of N/m^3.

Density of Ideal Gases

- ✓ **Equation of State**: equation for the relationship between pressure, temperature, and density.

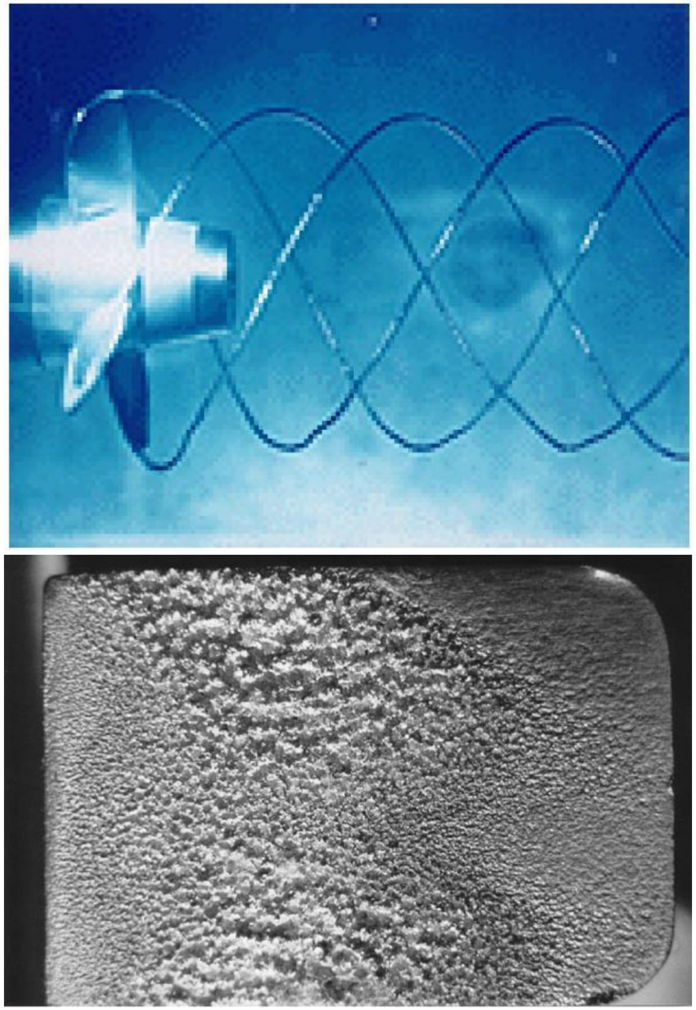
- ✓ The simplest and best-known equation of state is the ideal-gas equation.

$$\mathbf{P\,v = R\,T} \qquad \text{or} \qquad \mathbf{P = \rho R\,T}$$

- ✓ Ideal-gas equation holds for most gases.

- ✓ However, **dense gases such as water vapor and refrigerant vapor should not be treated as ideal gases**. Tables should be consulted for their properties.

Vapor Pressure and Cavitation



- **Vapor Pressure** P_v is defined as ***the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature***
- If P drops below P_v, liquid is locally vaporized, creating cavities of vapor
- Vapor cavities collapse when local P rises above P_v
- Collapse of cavities is a violent process which can damage machinery
- Cavitation is noisy, and can cause structural vibrations.

Energy and Specific Heats

- ✓ Total energy *E* is comprised of numerous forms**:** **thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear**

- ✓ Units of energy are *joule* (*J*) or *British thermal unit* (BTU).
- ✓ Microscopic energy
  - ➢ **Internal energy** *u* is for a non-flowing fluid and is due to molecular activity
  - ➢ **Enthalpy** h=u+Pv is for a flowing fluid and includes flow energy (Pv)

- ✓ Macroscopic energy
  - ➢ **Kinetic energy** $ke=V^2/2$
  - ➢ **Potential energy** $pe=gz$

- ✓ In the absence of electrical, magnetic, chemical, and nuclear energy, the total energy is $e_{flowing}=h+V^2/2+gz$.

Coefficient of Compressibility

- ✓ How does fluid volume change with *P* and *T*?
- ➢ Fluids expand as *T* increases or *P* deceases
- ➢ Fluids contract as *T* decreases or *P* increases

- ✓ Need fluid properties that relate volume changes to changes in *P* and *T*.
  - ➢ Coefficient of compressibility

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$$

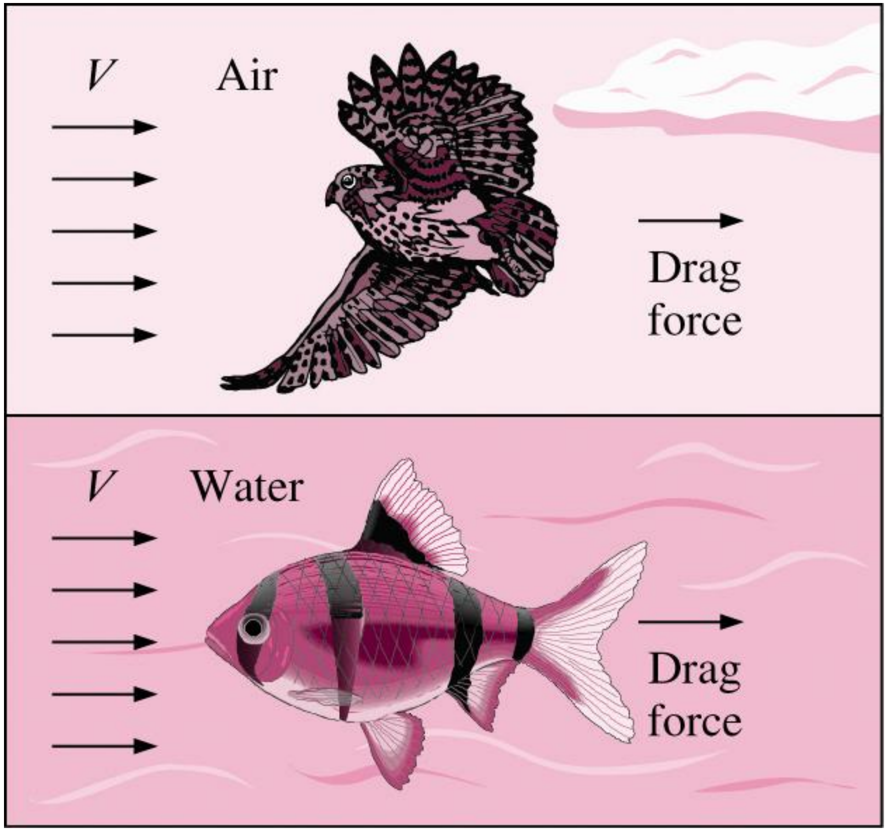
  - ➢ Coefficient of volume expansion

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

- ✓ Combined effects of *P* and *T* can be written as

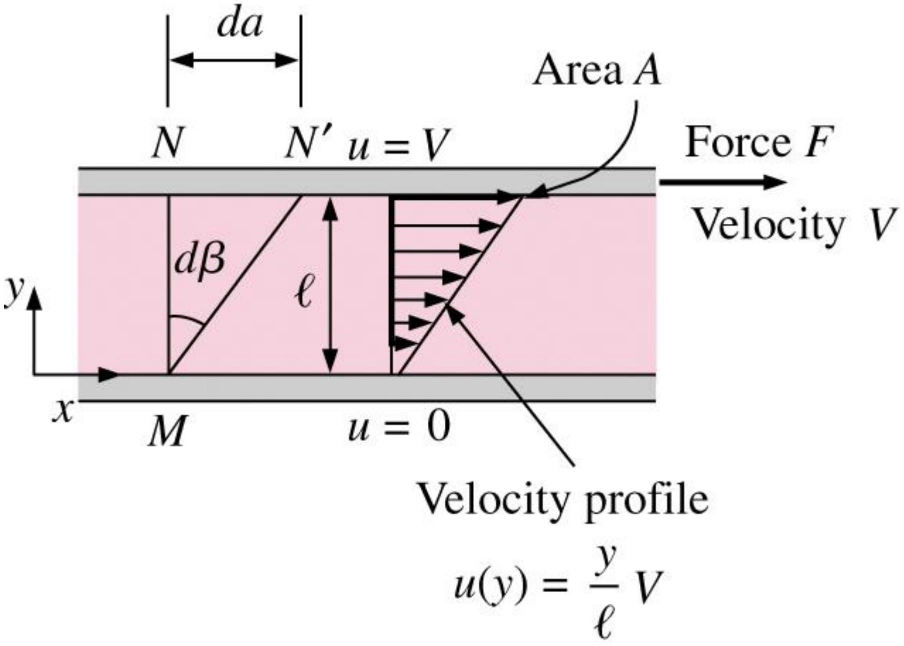
$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP$$

Viscosity



- **Viscosity** is a property that represents the internal resistance of a fluid to motion.
- The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity.

Viscosity



- To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates separated by a distance ℓ
- Definition of shear stress is $\tau = F/A$.
- Using the no-slip condition, $u(0) = 0$ and $u(\ell) = V$, the velocity profile and gradient are $u(y)= Vy/\ell$ and $du/dy=V/\ell$
- Shear stress for Newtonian fluid:

$$\boldsymbol{\tau = \mu \frac{du}{dy}}$$

- μ is the **dynamic viscosity** and has units of $kg/m{\cdot}s$, $Pa{\cdot}s$, or **poise**.

Newtonian and non-Newtonian fluids

We concluded:

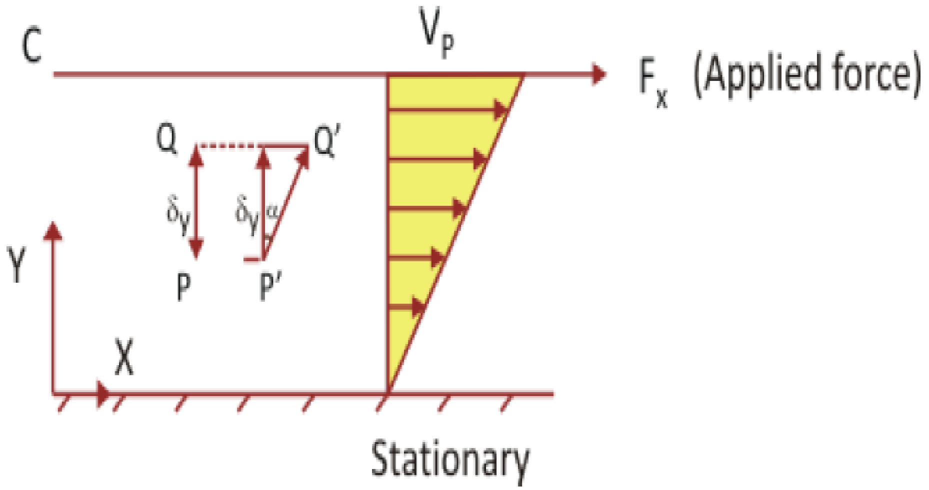
- ✓ Fluid **continues to deform** (or move) under the **application of a shear force**
- ✓ Fluid at **rest cannot sustain a shear stress**
- ✓ Fluid **resists deformation** by attaining an **equilibrium rate of deformation**.
- ✓ **Viscosity** is a property of the fluid which relates **its resistance to the applied shear force.**

Newton's law of viscosity: Newtonian fluids

- ✓ A class of fluids called ***Newtonian*** fluids such as **water and air** behaves as follows: "applied shear stress varies linearly with the rate of deformation".
- ✓ The mathematical representation is

$$\underset{\substack{\downarrow \\ \text{applied} \\ \text{stress}}}{\tau} = \underset{\substack{\downarrow \\ \text{Pa}-\text{s or } \frac{\text{N}-\text{s}}{\text{m}^2}}}{\mu} \underset{\substack{\downarrow \\ \text{Rate of strain} \\ \text{or deformation rate}(\text{s}^{-1})}}{\left(\frac{dV_x}{dy}\right)}$$

μ = Dynamic (or absolute) viscosity (Pa − s)



- ✓ It has been shown that
- ✓ For Newtonian fluid (w $\mu \equiv \mu\left(T, \frac{dv_x}{dy}\right)$ etc):

$$\mu \equiv \mu\,(T).$$

- ➢ For liquids, viscosity increases with temperature.
- ➢ For gases, viscosity decreases with temperature.

Non-Newtonian fluids

$$\mu \equiv \mu\left(\frac{dv_x}{dy}\right)^{n}.$$

Examples: Sugar solutions, Polymers

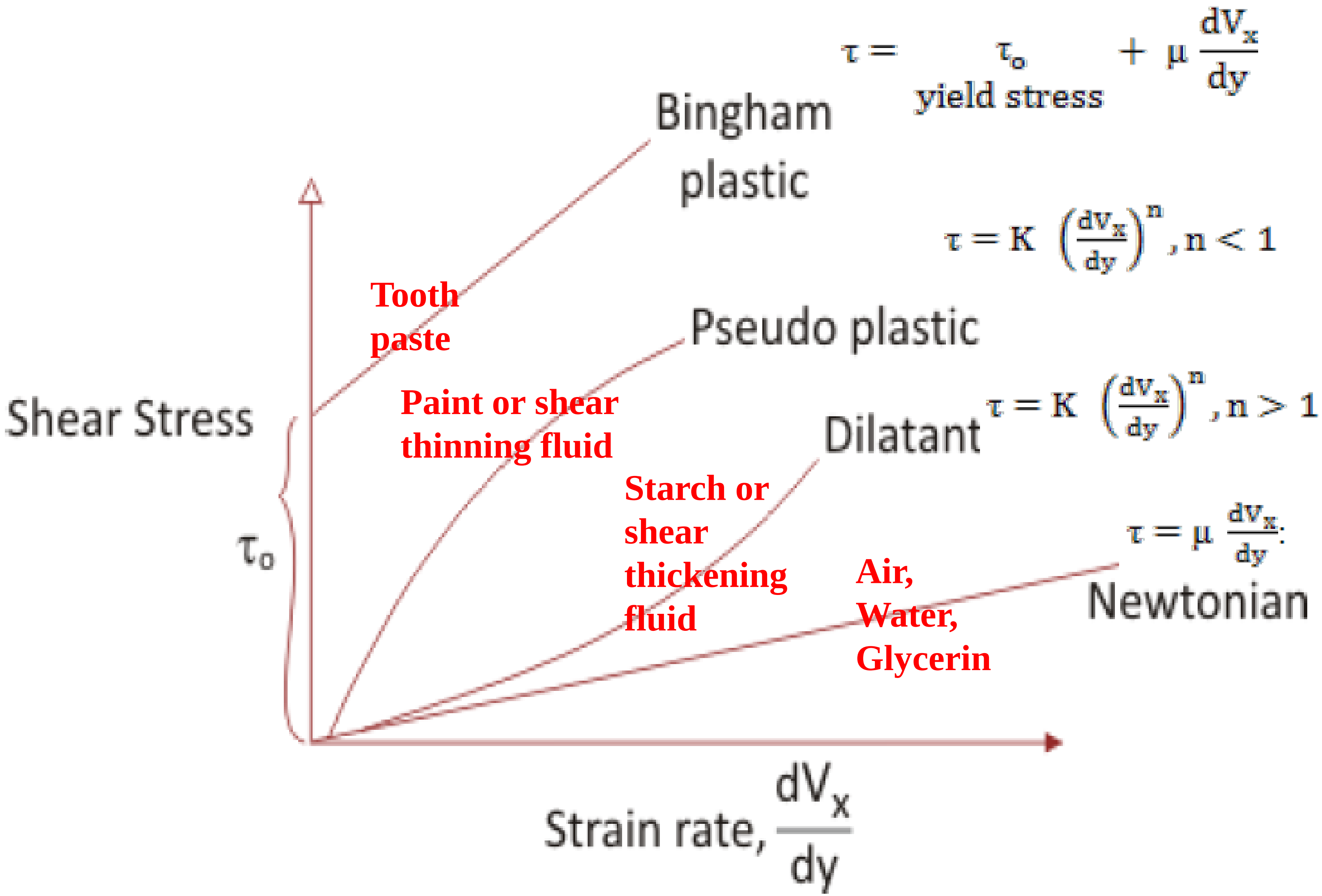
$$\boxed{\tau = m \left|\frac{dV_x}{dy}\right|^{n-1} \left(\frac{dV_x}{dy}\right)}$$

m is the flow consistency index
n is the flow behaviour index

Also known as **power–Law model**

$$m \left|\frac{dv_x}{dy}\right|^{n-1}$$

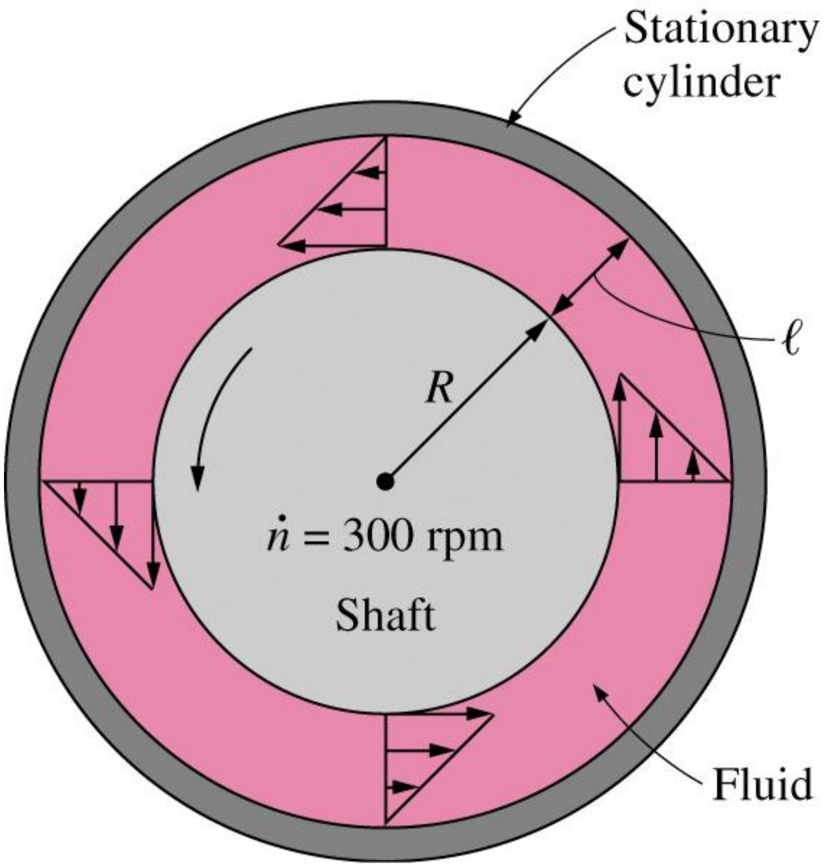
Apparent viscosity



Shear thickening fluids : Fluid starts ‘thickening' with increase in its apparent viscosity

Shear thinning fluids : Fluid starts ‘thinning' with decrease in its apparent viscosity

Viscometry



- ✓ How is viscosity measured? A rotating viscometer.
  - ➢ Two concentric cylinders with a fluid in the small gap ℓ.
  - ➢ Inner cylinder is rotating, outer one is fixed.
- ✓ Use definition of shear force:

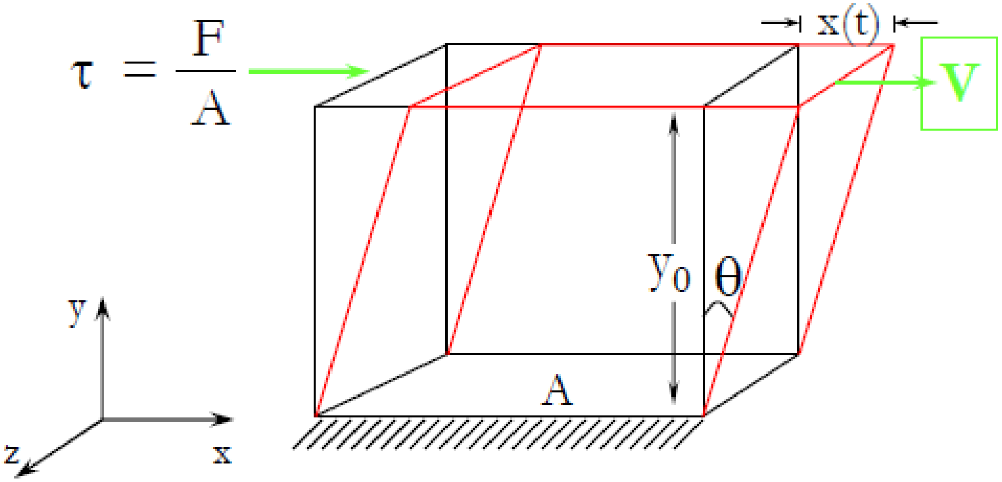
$$F = \tau A = \mu A \frac{du}{dy}$$

- ✓ If $\ell/R \ll 1$, then cylinders can be modeled as flat plates.
- ✓ Torque $T = FR$, and tangential velocity $V=\omega R$
- ✓ Wetted surface area $A=2pRL$.
- ✓ Measure T and ω to compute m

Fundamentals of Rheological Property Measurements

Simple Shear Deformation and Shear Flow

Shear Deformation



$$\tau = \frac{F}{A}$$

$$\text{Strain, } \gamma = \frac{x(t)}{y_0}$$

$$\text{Strain Rate, } \dot{\gamma} = \frac{V}{y_0} = \frac{1}{y_0}\frac{d\,x(t)}{d\,t}$$

$$\text{Viscosity, } \eta = \frac{\tau}{\dot{\gamma}}$$

$$\text{Shear Modulus, } G = \frac{\tau}{\gamma}$$

$$\dot{\gamma} = \frac{\Delta\gamma}{\Delta t}$$

Range of Rheological Material Behavior

- Rheology: The study of deformation and flow of matter *at specified conditions*.

Range of material behavior

Solid Like ---------- *Liquid Like*

(Ideal Solid ------------- Ideal Fluid)

Classical Extremes

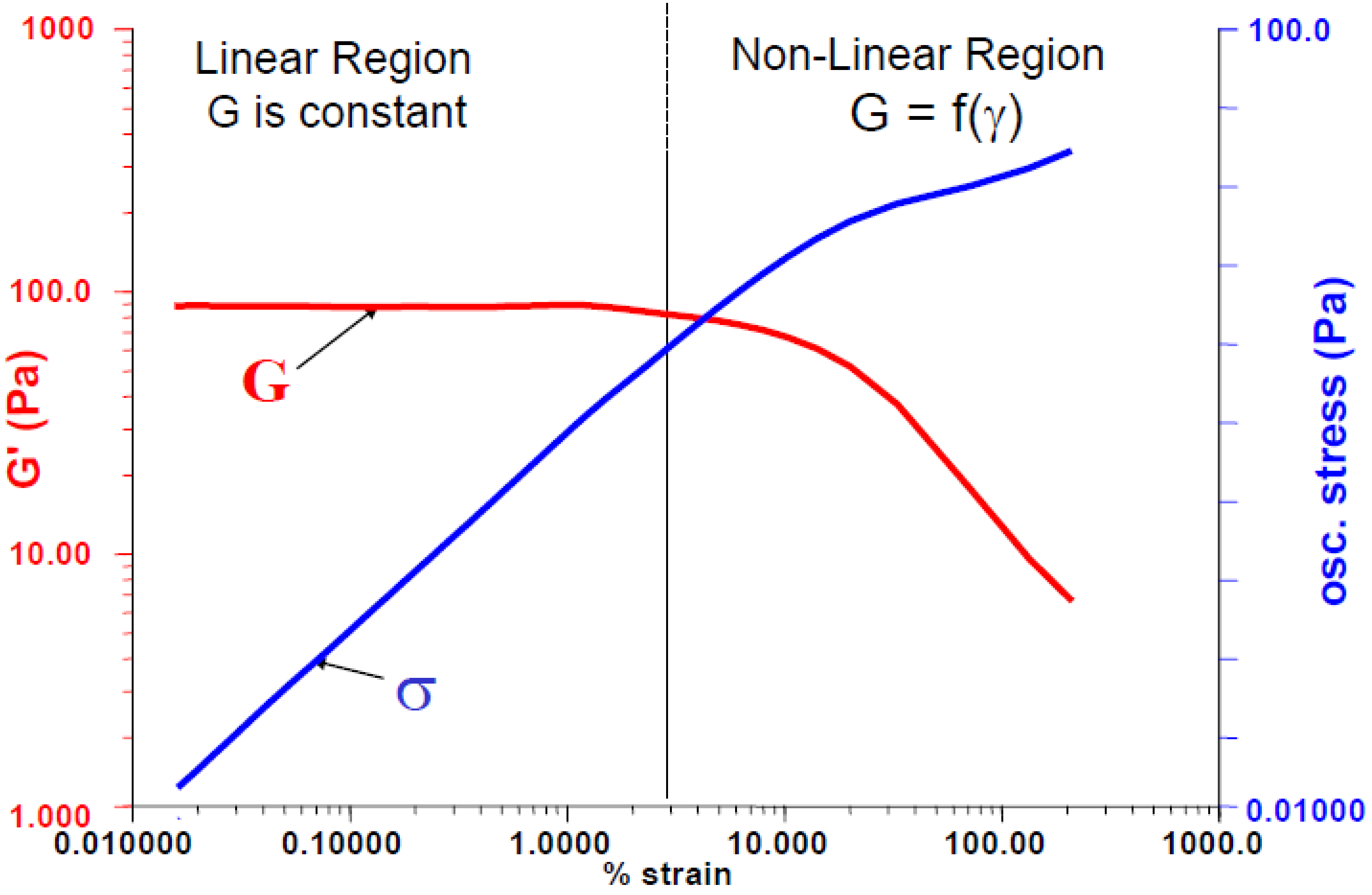
Classical Extremes: Elasticity

- 1678: Robert Hooke develops his "True Theory of Elasticity"
  - "The power of any spring is in the same proportion with the tension thereof."
  - Hooke's Law: $\tau = G \gamma$ or (Stress = G x Strain)

where G is the RIGIDITY MODULUS

Linear and Non-Linear Stress-Strain

Behavior of Solids

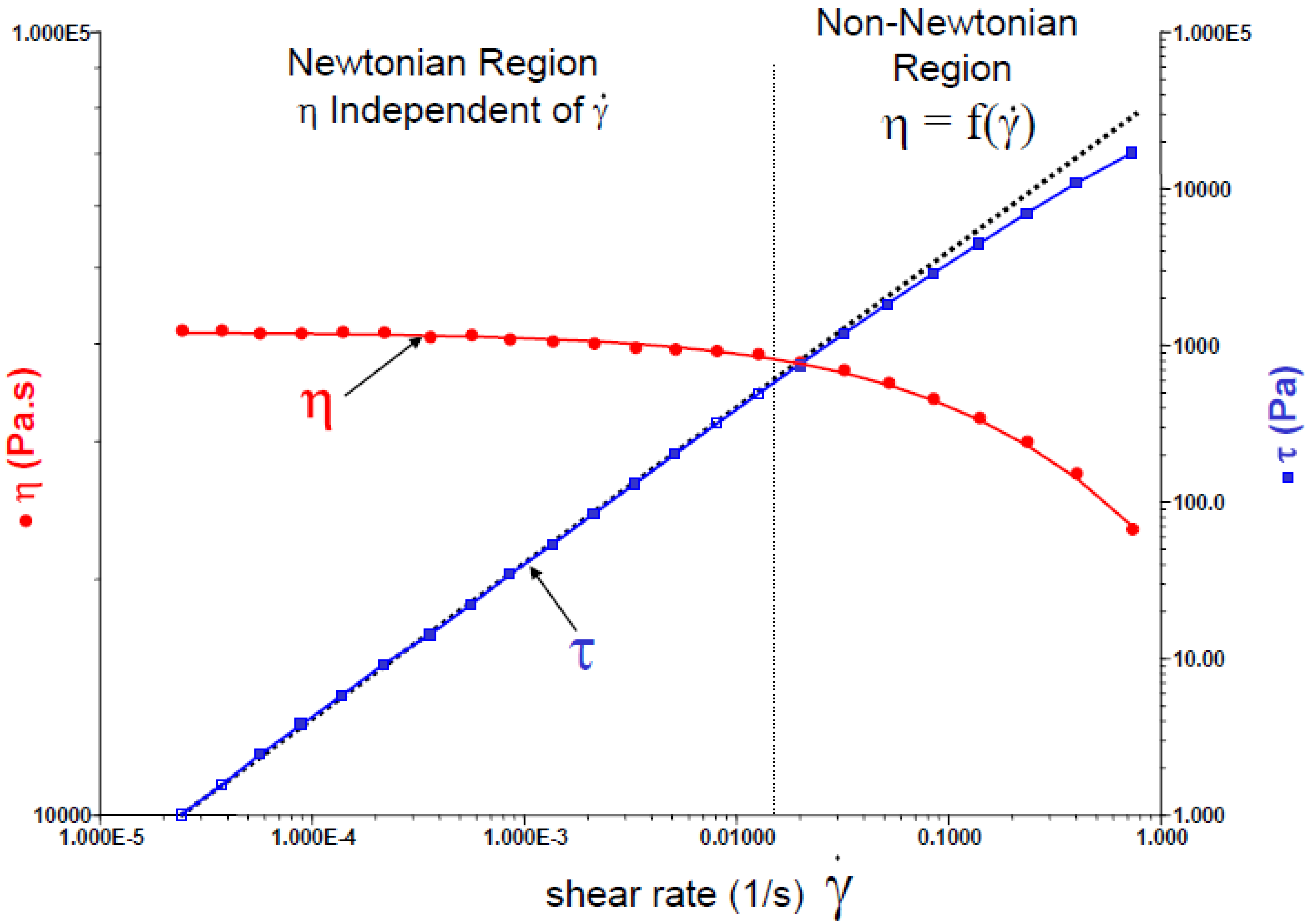


Classical Extremes: Viscosity

- 1687: Isaac Newton addresses liquids and steady simple shearing flow in his "*Principia*"
  - "The resistance which arises from the lack of slipperiness of the parts of the liquid, other things being equal, is proportional to the velocity with which the parts of the liquid are separated from one another."
  - Newton's Law: $\tau = \eta\dot{\gamma}$

where η is the Coefficient of Viscosity

Newtonian and Non-Newtonian Behavior of Fluids



PARAMETERS for Rheological Properties

Classical Extremes

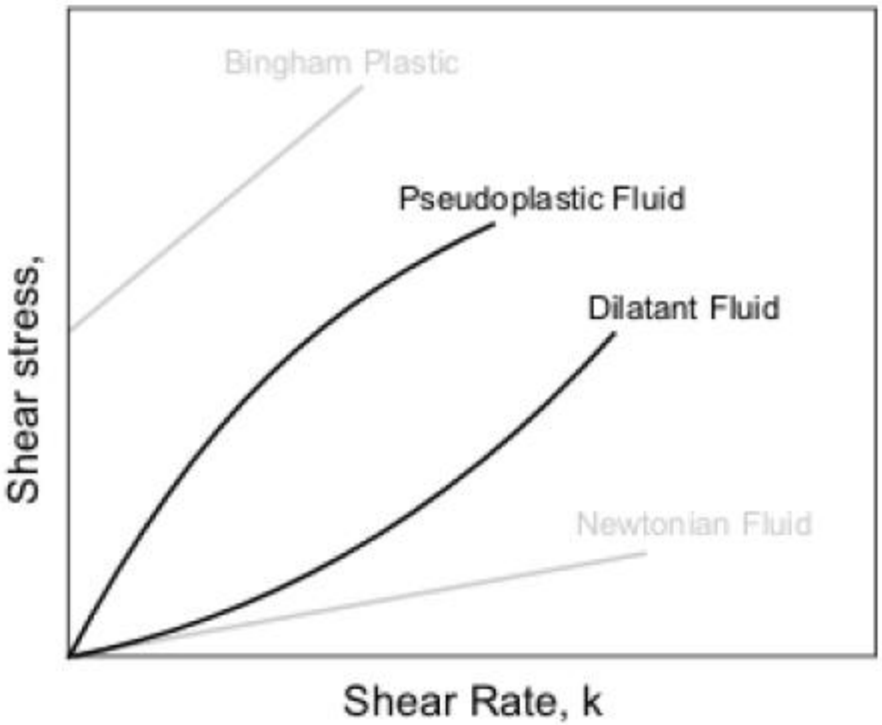
Ideal Solid	*-- [External Force] --*	*Ideal Fluid*
STEEL		**WATER**
Strong Structure		**Weak Structure**
Rigidity		**Fluidity**
Deformation		***Flow***
Retains/recovers form		**Losses form**
Stores Energy		**Dissipates Energy**
(Purely Elastic – R. Hooke, 1678)	*[Energy]*	*(Purely Viscous – I. Newton, 1687)*
ELASTICITY		**VISCOSITY**
Storage Modulus		**Loss Modulus**

REAL Behavior

Apparent Solid	*[Energy + time]*	*Apparent Fluid*

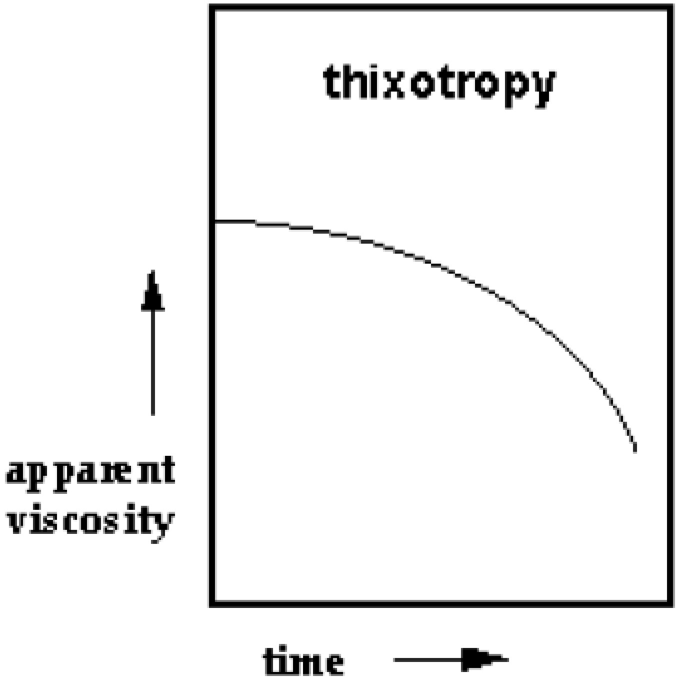
- viscoelastic materials -

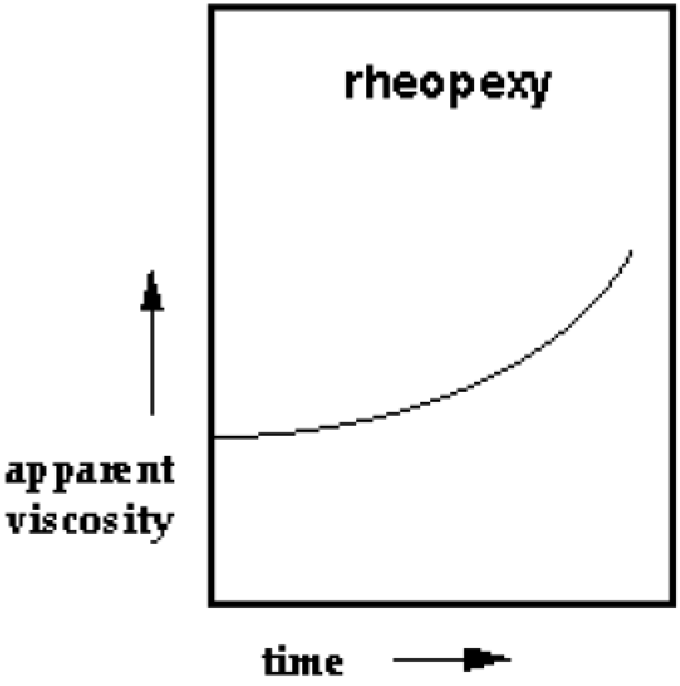
Stress-strain rate curve

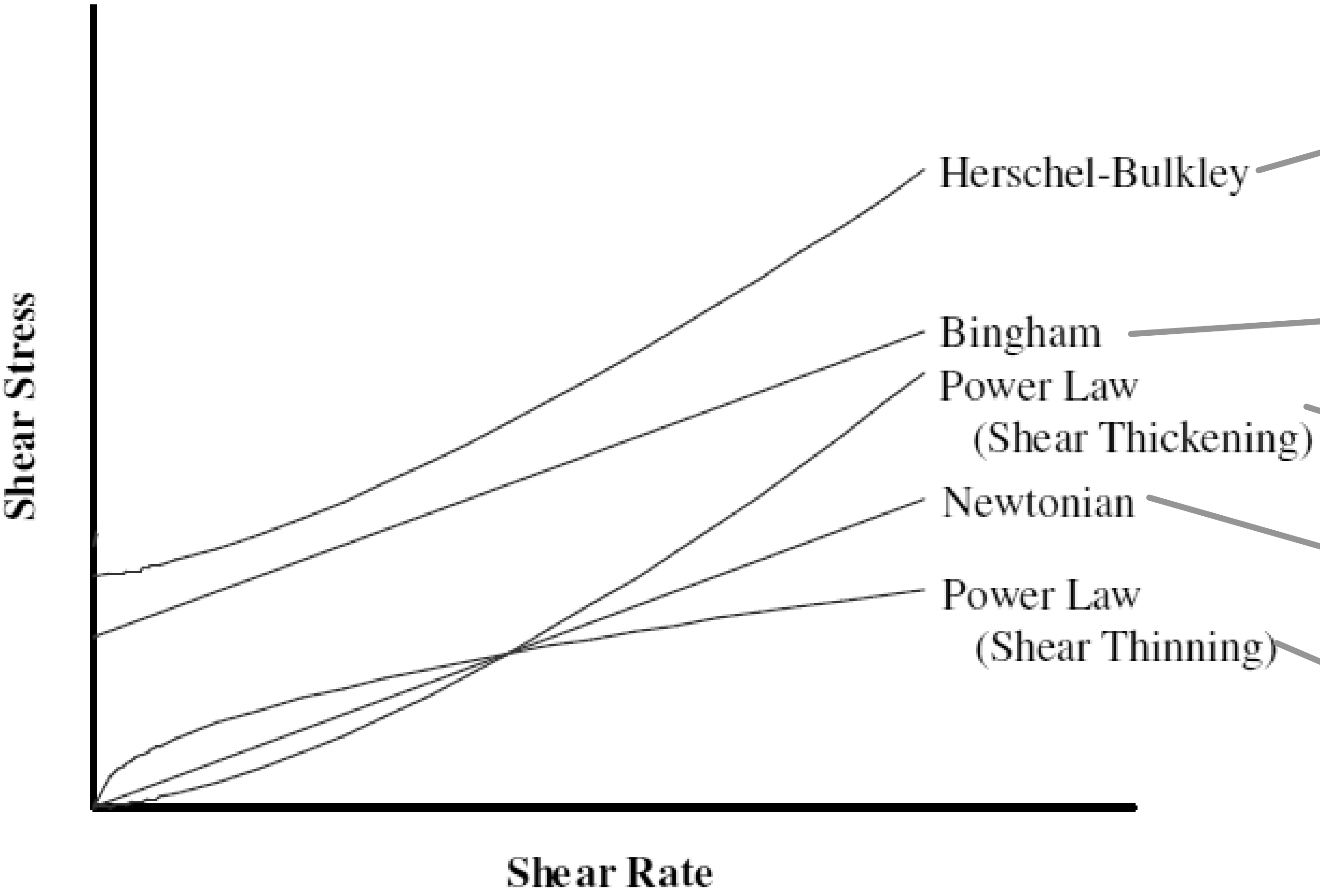


Dilatancy (shear thickening)
Plastic and Pseudoplastic (shear thinning)

apparent viscosity as a function of time







$$\tau = \tau_0 + a\dot{\gamma}^b$$

$$\tau = \tau_0 + \mu\dot{\gamma}$$

$$\tau = a\dot{\gamma}^b$$

$$\tau = \eta\dot{\gamma}$$

$$\tau = a\dot{\gamma}^b$$

Viscometer vs. Rheometer

- **Viscometer:** instrument that measures the viscosity of a fluid over a limited shear rate range
- **Rheometer**: instrument that measures:
  - *Viscosity over a wide range of shear rates, and…*
  - *Viscoelasticity of fluids, semi-solids and solids*

Constitutive Relations

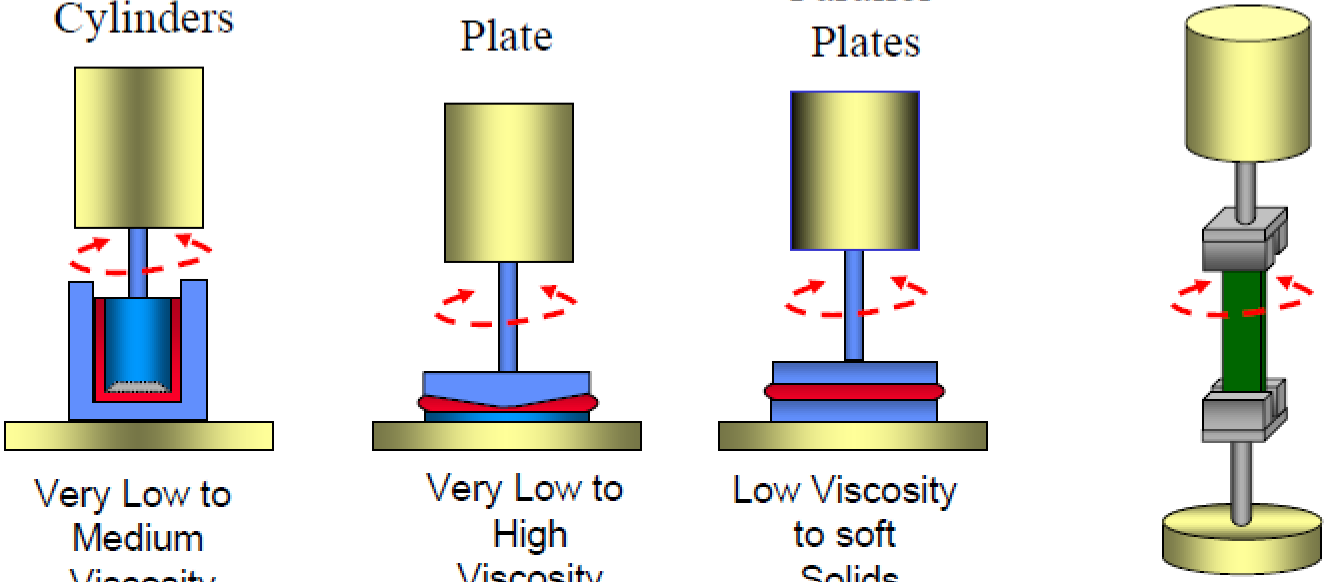
$$\frac{Stress}{Strain} = Modulus$$

$$\frac{Stress}{Shear\ rate} = \mathrm{Viscosity}$$

Experimental Methods/ Instruments

- Capillary viscometers
  - Cup
  - Glass
  - Extrusion rheometers
- Rotational rheometers
  - Parallel plates (disks)
  - Cone-and-plate
  - Couette
  - Brookfield viscometers
- Falling ball viscometers
- Extensional rheometers
- …

Measuring Systems - Geometries



Concentric Cylinders

Very Low to Medium Viscosity

Cone and Plate

Very Low to High Viscosity

Parallel Plates

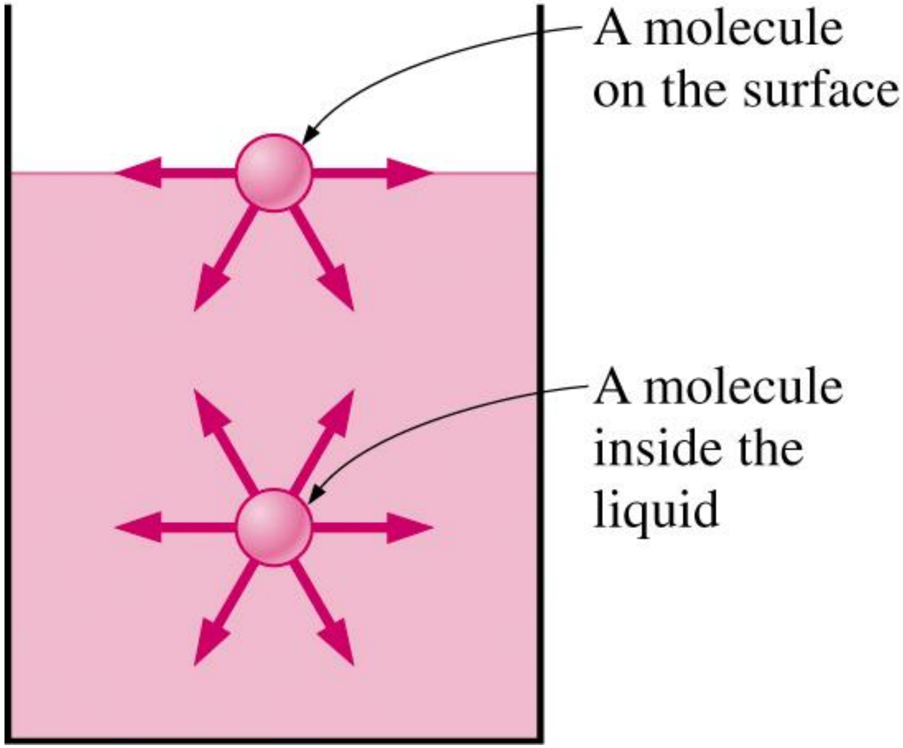
Low Viscosity to soft Solids

Rectangular Torsion

Soft to Rigid Solids

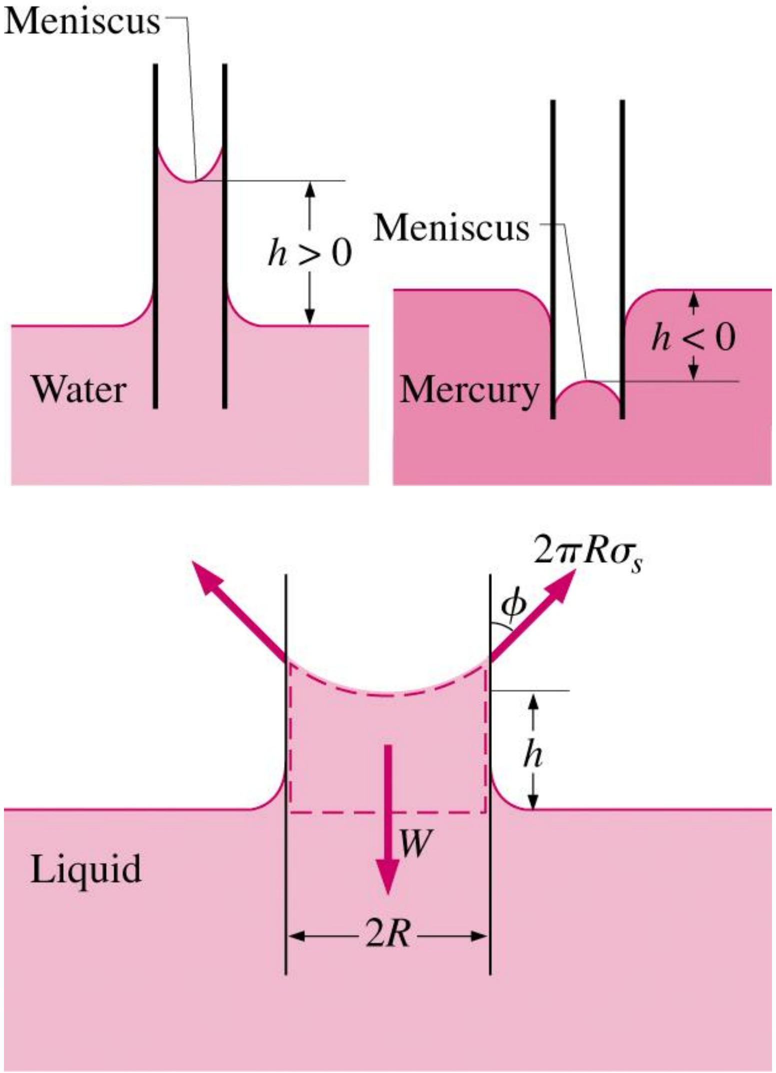
Decane → Water → Steel

Surface Tension



- ✓ Liquid droplets behave like small spherical balloons filled with liquid, and the surface of the liquid acts like a stretched elastic membrane under tension.
- ✓ The pulling force that causes this is
  - ➢ due to the attractive forces between molecules
  - ➢ called **surface tension** s_s.
- ✓ Attractive force on surface molecule is not symmetric.
- ✓ Repulsive forces from interior molecules causes the liquid to minimize its surface area and attain a spherical shape.

Capillary Effect



- **Capillary effect** is the rise or fall of a liquid in a small-diameter tube.
- The curved free surface in the tube is call the **meniscus**.
- Water meniscus curves up because water is a ***wetting fluid.***
- Mercury meniscus curves down because mercury is a ***non-wetting fluid***.
- Force balance can describe magnitude of capillary rise.