Lecture 4
Binary Arithmetic

Binary Arithmetic

- Addition
- Subtraction
- Complements – 1's and 2's

Binary Addition

(a)
$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array}$$

(b)
$$\begin{array}{r} 0 \\ +\,1 \\ \hline 1 \end{array}$$

(c)
$$\begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array}$$

(d)
$$\begin{array}{r} 1 \\ +\,1 \\ \hline 1\,0 \end{array}$$

Carry Bit

Binary Addition Examples

(a)
```
   1011
+  1100
-------
  10111
```

(b)
```
   1010
+   100
-------
   1110
```

(c)
```
   1011
+   101
-------
  10000
```

(d)
```
    101
+  1001
-------
   1110
```

(e)
```
   10011001
+    101100
-----------
   11000101
```

Binary Complement

(1s Complement) Operation

1 → 0

0 → 1

Example

1 1 0 0 1 0 1 1 0

0 0 1 1 0 1 0 0 1

Two's Complement

The Two's complement of a binary number is obtained by first complementing the number and then adding 1 to the result.

1001110

0110001 ⇐ **One's Complement**

+ 1

0110010 ⇐ **Two's Complement**

Binary Subtraction

Binary subtraction is implemented by adding the Two's complement of the number to be subtracted.

Example

$$\begin{array}{r} 1101 \\ -1001 \end{array} \qquad\qquad \begin{array}{r} 1101 \\ +0111 \\ \hline 10100 \end{array}$$

Two's complement of 1001

If there is a carry then it is ignored. Thus, the answer is 0100.

Basic Digital Arithmetic

- Signed Binary Number: A binary number of fixed length whose sign (+/-) is represented by one bit (usually MSB) and its magnitude by the remaining bits
- Unsigned Binary Number: A binary number of fixed length whose sign is not specified by a bit. All bits are magnitude and the sign is assumed +.

Signed Binary Numbers

- Sign Bit: A bit (usually the MSB) that indicates whether a number is positive(=0) or negative (=1).
- Magnitude Bits: The bits of a signed binary number that tell how large it is in value.
- True Magnitude Form: A form of signed binary whose magnitude bits are the TRUE binary form (not complements).

Signed Binary Numbers II

- 1s Complement: A form of signed binary in which negative numbers are created by complementing all bits.
- 2s Complement: A form of signed binary in which the negative numbers are created by complementing all the bits and adding a 1 (1s Complement +1).

Unsigned Binary Arithmetic

- Sum: Result of an Addition Operation of two (or more) binary numbers (operands).
- Carry: A digit (or bit) that is carried over to the next most significant bit during an N Bit addition operation.
- The carry bit is a 1 if the result was too large to be expressed in N bits.

Basic Rules (Unsigned)

- One Bit Unsigned Addition

0 + 0 =	0 0
1 + 0 =	0 1
1 + 1 =	1 0

1 + 1 + 1 = 1 1

CIN A B COUT SUM

True Magnitude Form

- 5 Bit Numbers Negative = S=1

+25 = 0 $\boxed{11001}$

-25 = 1 $\boxed{11001}$

-12 = 1 $\boxed{01100}$

+12 = 0 $\boxed{01100}$

Note sign bit = MSB = S = 0

Same as +25 with S=1

True magnitude

2's complement of a binary number:

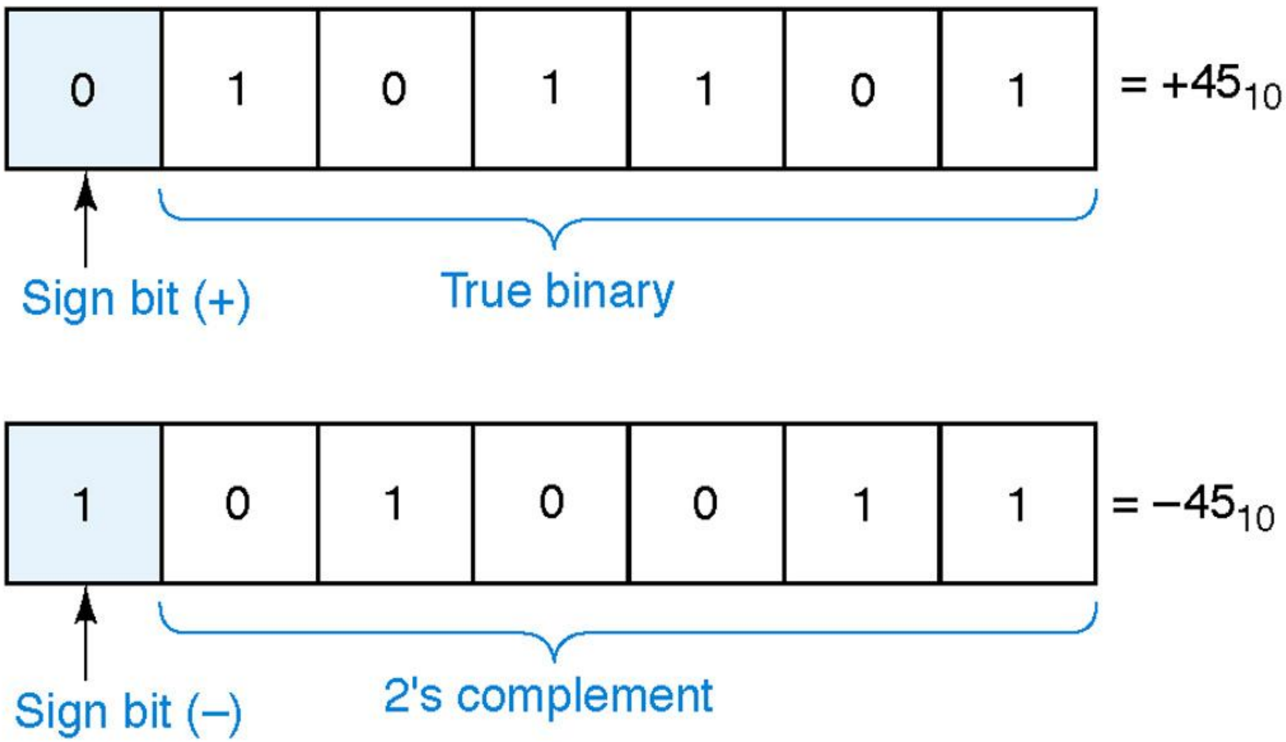
- Take the 1's complement of the number
- Add 1 to the least-significant-bit position

$$\begin{array}{rl} 101101 & \text{binary equivalent of } 45 \\ 010010 & \text{complement each bit to form 1's complement} \\ + \qquad 1 & \text{add 1 to form 2's complement} \\ \hline 010011 & \text{2's complement of original binary number} \end{array}$$

Representing signed numbers using 2's complement form

- If the number is positive, the magnitude is represented in its positional-weighted binary form, and a sign bit of 0 is placed in front of the MSB.
- If the number is negative, the magnitude is represented in its 2's complement form, and a sign bit of 1 is placed in front of the MSB.

example



0
1
0
1
1
0
1
= +45₁₀
Sign bit (+)
True binary
1
0
1
0
0
1
1
= −45₁₀
Sign bit (–)
2's complement

Example

- Represent each of the following signed decimal numbers as a signed binary number in the 2's-complement system. Use a total of five bits including the sign bit.

  (a) +13 (b) –9 (c) +3 (d) –2 (e) -8

Addition in the 2's-complement system

- Case I: Two Postive Numbers.

+9	$\rightarrow$	0	1001	(augend)
+4	$\rightarrow$	0	0100	(addend)
		0	1101	(sum = +13)

Sign bits

Addition, cont.

- Case II: Positive Number and Smaller Negative Number

$$
\begin{array}{lr}
+9 \rightarrow & 0\ 1001 \text{ (augend)} \\
-4 \rightarrow & \underline{1\ 1100 \text{ (addend)}} \\
 & 1\ \ 0\ 0101
\end{array}
$$

Sign bits

This carry is disregarded; the result is 0101(sum=+5)

Addition, cont.

- Case III: Positive Number and Larger Negative Number

Negative sign bit

$$\begin{array}{rl} -9 \rightarrow & 10111 \\ \underline{+4 \rightarrow} & \underline{00100} \\ & 11011 \quad \text{(sum = -5)} \end{array}$$

Addition, cont.

- Case IV: two negative Numbers

$-9 \rightarrow 10111$

$-4 \rightarrow 11100$

$1\ 10011$

Sign bit

This carry is disregarded; the result is 10011(sum =-13)

Negative Result Example

- 2s Complement Negative Result (65-80)

+65 =	0 100 0001	100 0001
-80 =	1 101 0000 (2s C.)	+ 011 0000
		111 0001
	Invert	000 1111
	Add 1	+ 1
	Final Result = -15	0000 1111 = 15(Neg.)

Addition, cont.

- Case V: Equal and Opposite Numbers

$$
\begin{array}{llll}
-9 \rightarrow & 1 & 0111 \\
\underline{+9 \rightarrow} & \underline{0} & \underline{1001} \\
0 \quad 1 & & 0000
\end{array}
$$

Disregard; the result is 0000(sum = +0)

Subtraction in the 2’s-complement System

- The procedure for subtracting one binary number(the subtrahend) from another binary number(the minuend)
  - Negate the subtrahend. This will change the subtrahend to its equivalent value of opposite sign.
  - Add this to the minuend. The result of this addition will represent the difference between the subtrahend and the minuend.

Addition and Subtraction of BCD and Excess-3 Code

Unsigned Numbers BCD Addition

Use binary arithmetic to add the BCD digits:

8	1000	Eight	
+5	+0101	Plus 5	
13	1101	is 13 (> 9)	

$$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$$ OK (< 9)

If result is > 9, it must generate a carry and be corrected!

To correct the digit, add 0110 in the result.

8	1000	Eight
+5	+0101	Plus 5
13	1101	13 (is > 9)
	+0110	so add 6 (always, for results > 9)
	carry = 1 0011	giving 3 + carry
	0001 | 0011	Final answer (two digits)

We try to avoid subtraction!
Replacing it with addition!

The adder circuit utilizes the resulting carry bit by sending it as carry-in to the next digit

Add 2905_{BCD} to 1857_{BCD} showing carries and digit corrections.

1010	1	0	1	0
1857	0001	1000	0101	0111
+2905	+ 0010	1001	0000	0101
	0100 4<9 → +0	10001 17>9 → +6	0110 6<9 → +0	1100 12>9 → +6
Add 6 or 0	+ 0000	0110	0000	0110
4762	0100	1 0111	0110	1 0010

Excess-3 Code

A BCD Code formed by adding 3 (0011) to its true 4-bit binary value.

 Excess-3 is a self-complementing code:

 A negative code equivalent can be found by inverting the binary bits of the positive code

 Inverting the bits of the Excess-3 digit yields 9's Complement of the decimal equivalent.

Example : Excess -3 code of decimal 4 is 0111. (0100 + 0011 = 0111)

(4) = 0111

(-4) = 1000 (inverting the bits) which is Excess -3 code of decimal 5.

It is 9's complement of the decimal equivalent. (9 – 4 = 5)

Excess-3 Examples

- 3 = 0011 + 0011 = 0110 = 6 in E3.
- 1 = 0001 + 0011 = 0100 = 4 in E3.
- If we complement 1 = 1011 in E3, this is the code for an 8.
- 9's Complement of 1 = (9 – 1) = 8 (SelfComplement)

Assignment- 2

- Perform addition and subtraction using 2’s complement:-
- 10100001
- 10000111