Lecture 10
Standard Forms

STANDARD FORMS

SOP AND POS

- Boolean expressions can be manipulated into many forms.

- Some standardized forms are required for Boolean expressions to simplify communication of the expressions.

    - **Sum-of-products (SOP)**

        - Example:

$$F(A, B, C, D) = AB + \bar{B}C\bar{D} + AD$$

    - **Products-of-sums (POS)**

        - Example:

$$F(A, B, C, D) = (A + B)(\bar{B} + C + \bar{D})(A + D)$$

Minterms and Maxterms

➢ MINTERMS AND MAXTERMS:

n binary variables can be combined to form 2^n terms (AND terms), called *minterms* or standard products.

In a similar fashion, n binary variables can be combined to form 2^n terms (OR terms), called *maxterms* or standard sums.

* Note that each maxterm is the complement of its corresponding minterm and vice versa.

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Minterms and Maxterms (continued)

Minterms and Maxterms for Three Binary Variables

x y z	Minterms		Maxterms	
0 0 0	$x'y'z'$	m_0	$x+y+z$	M_0
0 0 1	$x'y'z$	m_1	$x+y+z'$	M_1
0 1 0	$x'yz'$	m_2	$x+y'+z$	M_2
0 1 1	$x'yz$	m_3	$x+y'+z'$	M_3
1 0 0	$xy'z'$	m_4	$x'+y+z$	M_4
1 0 1	$xy'z$	m_5	$x'+y+z'$	M_5
1 1 0	xyz'	m_6	$x'+y'+z$	M_6
1 1 1	xyz	m_7	$x'+y'+z'$	M_7

STANDARD FORMS

MINTERMS

The following table gives the minterms for a **three-input** system

A	B	C	m_0 $\overline{A}\overline{B}\overline{C}$	m_1 $\overline{A}\overline{B}C$	m_2 $\overline{A}B\overline{C}$	m_3 $\overline{A}BC$	m_4 $A\overline{B}\overline{C}$	m_5 $A\overline{B}C$	m_6 $AB\overline{C}$	m_7 ABC
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

STANDARD FORMS

SUM OF MINTERMS

- **Sum-of-minterms** standard form expresses the Boolean or switching expression in the form of a **sum of products** using **minterms**.
  - For instance, the following Boolean expression using minterms

$$F(A, B, C) = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C$$

could instead be expressed as

$$F(A, B, C) = m_0 + m_1 + m_4 + m_5$$

or more compactly

$$F(A, B, C) = \sum m(0, 1, 4, 5) = one\text{-}set(0, 1, 4, 5)$$

STANDARD FORMS
MAXTERMS

- The following table gives the maxterms for a **three-input** system

A	B	C	M_0 $A+B+C$	M_1 $A+B+\overline{C}$	M_2 $A+\overline{B}+C$	M_3 $A+\overline{B}+\overline{C}$	M_4 $\overline{A}+B+C$	M_5 $\overline{A}+B+\overline{C}$	M_6 $\overline{A}+\overline{B}+C$	M_7 $\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

STANDARD FORMS

PRODUCT OF MAXTERMS

- **Product-of-maxterms** standard form expresses the Boolean or switching

  expression in the form of **product of sums** using **maxterms**.

  - For instance, the following Boolean expression using maxterms

  $$F(A, B, C) = (A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C})$$

  could instead be expressed as

  $$F(A, B, C) = M_1 \cdot M_4 \cdot M_7$$

  or more compactly as

  $$F(A, B, C) = \prod M(1, 4, 7) = \textit{zero-set}(1, 4, 7)$$

STANDARD FORMS

MINTERM AND MAXTERM EXP.

Given an arbitrary Boolean function, such as

$$F(A, B, C) = AB + \overline{B}(\overline{A} + \overline{C})$$

how do we form the canonical form for:

- **sum-of-minterms**
  - Expand the Boolean function into a sum of products. Then take each term with a missing variable X and **AND** it with $X + \overline{X}$.
- **product-of-maxterms**
  - Expand the Boolean function into a product of sums. Then take each factor with a missing variable X and **OR** it with $X\overline{X}$.
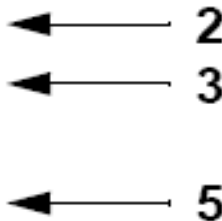
STANDARD FORMS

FORMING SUM OF MINTERMS

- Example

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

$$= \sum m(0, 1, 4, 6, 7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as 1s in Truth Table

STANDARD FORMS
FORMING PROD OF MAXTERMS

- Example

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$= (A + \bar{B})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C}) \qquad \text{(using distributivity)}$$

$$= (A + \bar{B} + C\bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

$$= (A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

$$= \prod M(2, 3, 5)$$

A	B	C	F	
0	0	0	1	
0	0	1	1	
0	1	0	0	← 2
0	1	1	0	← 3
1	0	0	1	
1	0	1	0	← 5
1	1	0	1	
1	1	1	1	

Maxterms listed as
0s in Truth Table

STANDARD FORMS
CONVERTING MIN AND MAX

- Converting between sum-of-minterms and product-of-maxterms

  - The two are complementary, as seen by the truth tables.

  - To convert interchange the $\sum$ and $\prod$, then use missing terms.

    - Example: The example from the previous slides

$$F(A, B, C) = \sum m(0, 1, 4, 6, 7)$$

  is re-expressed as

$$F(A, B, C) = \prod M(2, 3, 5)$$

  where the numbers 2, 3, and 5 were missing from the minterm representation.

S minterms and P maxterms

- Given the truth table, express F_1 in sum of minterms

x	y	z	F_1	F_2
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F_1(x, y, z) = \Sigma(1,4,5,6,7) = m_1 + m_4 + m_5 + m_6 + m_7$$

$$= (x'\,y'\,z) + (xy'\,z') + (xy'\,z) + (xyz') + (xyz)$$

- Find F_2

Sminterms and P maxterms

- Repeat for product of maxterms.

x	y	z	F_1	F_2
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F_1(x, y, z) = \prod(0,2,3) = M_0 \cdot M_2 \cdot M_3$$

$$= (x + y + z)(x + y' + z)(x + y' + z')$$

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Sminterms and P maxterms

Express the Boolean function $F = x + y'z$ in a sum of minterms, and then in a product of Maxterms.

$$x = x(y + y') = xy + xy'$$

$$xy = xy(z + z') = xyz + xyz'$$

$$xy' = xy'(z + z') = xy'z + xy'z'$$

$$y'z = y'z(x + x') = xy'z + x'y'z$$

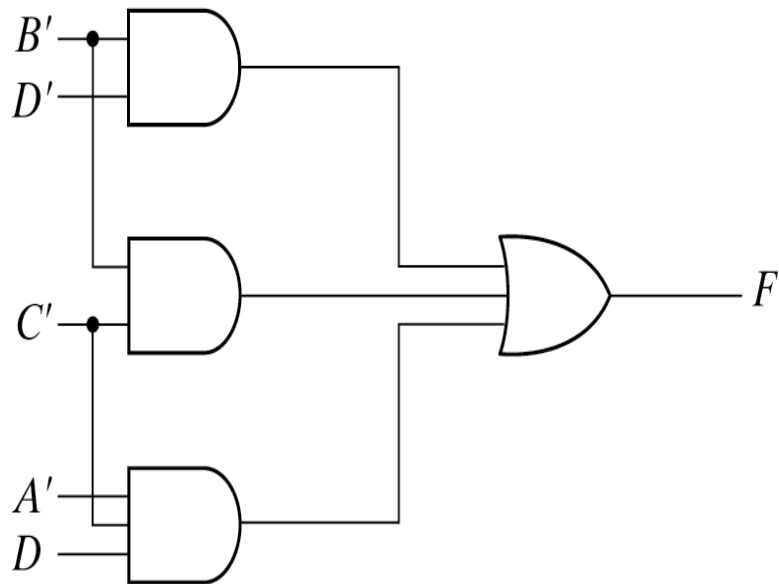Adding all terms and excluding recurring terms:

$$F(x, y, z) = x'y'z + xy'z' + xy'z + xyz' + xyz \quad \textbf{(SOP)}$$

$$F(x, y, z) = m_1 + m_4 + m_5 + m_6 + m_7 = \Sigma(1,4,5,6,7)$$
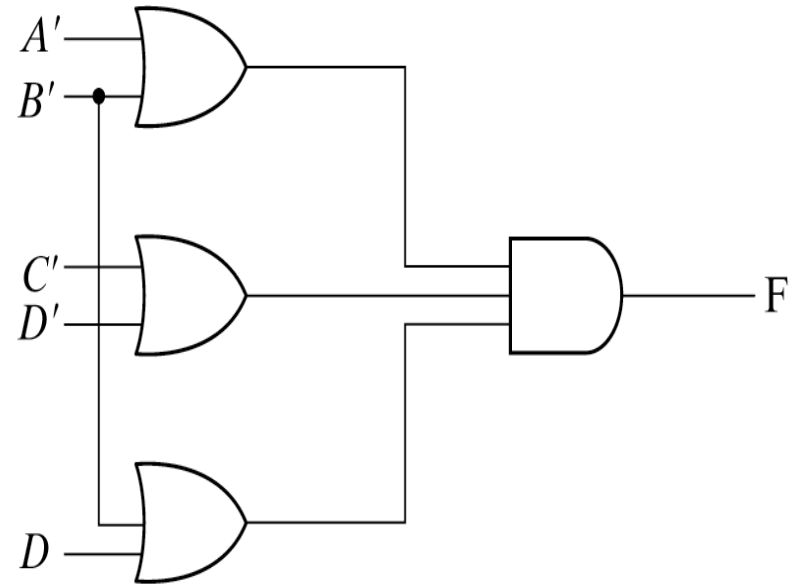
Product of maxterms (POS)?

SOP and POS gate implementation

SUM OF PRODUCT (SOP)   PRODUCT OF SUM (POS)

(a) $F = B'D' + B'C' + A'C'D$ (b) $F = (A' + B')(C' + D')(B' + D)$

Fig. 3-15 Gate Implementation of the Function of Example 3-8

Assignment 10

- Convert the following to SOP and POS Form as a function of ABC :

F(A,B,C) = 110 , 111, 101 ,001