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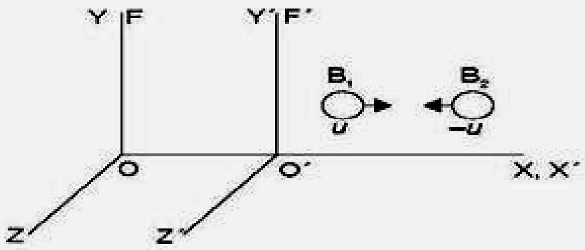
Variation of mass with velocity

In classical mechanics, mass of a particle is considered to be a constant quantity and independent of its velocity. However, in relativistic mechanics, like length and time, the mass also depends on its velocity. Consider two frames of references F and F′ such that F′ is moving with a constant velocity v relative to F in the positive direction of X. Suppose, two similar elastic balls B_1 and B_2 each having the same mass m approach each other in the frame F′ with equal speeds (u and $-u$) and collide with each other in such a way that they coalesce into one body. By applying the law of conservation of linear momentum, we have,



Momentum of ball B_1 + momentum of ball B_2 = momentum of coalesced body

$$(mu) + (-mu) = 0$$

Thus, the coalesced body must be at rest in frame F′. Now, let us consider the collision with regard to the frame of reference F, where u_1 and u_2 are the velocities of the balls. Then, according to Lorentz velocity transformations

$$u_1 = \frac{u+v}{1+uv/c^2} \quad \text{(i)}$$

$$u_2 = \frac{-u+v}{1-uv/c^2} \quad \text{(ii)}$$

After an inelastic collision, the coalesced body moves with the velocity of frame F′, (as it remains at rest in F′). Thus, v is the observed velocity in frame F. Let mass of the ball B_1 moving with velocity u_1 is m_1 and that of ball B_2 moving with velocity u_2 is m_2 in the frame of reference F. By applying conservation of linear momentum, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \text{(iii)}$$

On substituting u_1 and u_2 from Eqs. (i) and (ii) into Eq. (iii), we have

$$m_1\left[\frac{u+v}{1+uv/c^2}\right] + m_2\left[\frac{-u+v}{1-uv/c^2}\right] = (m_1 + m_2)v$$

$$m_1\left[\frac{u+v}{1+uv/c^2}\right] - m_1v = m_2v - m_2\left[\frac{-u+v}{1-uv/c^2}\right]$$

$$m_1\left[\frac{u+v}{1+uv/c^2} - v\right] = m_2\left[v - \frac{-u+v}{1-uv/c^2}\right]$$

$$m_1\left[\frac{u(1-v^2/c^2)}{1+uv/c^2}\right] = m_2\left[\frac{u(1-v^2/c^2)}{1+uv/c^2}\right]$$

$$\text{or} \quad \frac{m_1}{m_2} = \frac{1 + uv / c^2}{1 - uv / c^2} \tag{iv}$$

Now from Eq. (i),

$$1 - \frac{u_1^{\ 2}}{c^2} = 1 - \frac{\left(\frac{u+v}{c}\right)^2}{\left(1 + \frac{uv}{c^2}\right)^2} = \frac{(1 - u^2 / c^2)(1 - v^2 / c^2)}{(1 + uv / c^2)^2} \tag{v}$$

Similarly, we can write

$$1 - \frac{u_2^{\ 2}}{c^2} = \frac{(1 - u^2 / c^2)(1 - v^2 / c^2)}{(1 - uv / c^2)^2} \tag{vi}$$

On dividing Eq. (vi) by Eq .(v),we have

$$\frac{1 - u_2^{\ 2} / c^2}{1 - u_1^{\ 2} / c^2} = \frac{(1 + uv / c^2)^2}{(1 - uv / c^2)^2}$$

$$\frac{\sqrt{1 - u_2^{\ 2} / c^2}}{\sqrt{1 - u_1^{\ 2} / c^2}} = \frac{1 + uv / c^2}{1 - uv / c^2} \tag{vii}$$

Thus, from Eqs.(iv) and (vii), we have,

$$\frac{m_1}{m_2} = \frac{\sqrt{1 - u_2^{\ 2} / c^2}}{\sqrt{1 - u_1^{\ 2} / c^2}}$$

$$m_1\left[\sqrt{1 - u_1^{\ 2} / c^2}\right] = m_2\left[\sqrt{1 - u_2^{\ 2} / c^2}\right] \tag{viii}$$

From Eq. (viii), it is clear that the left hand side and right hand side are independent of one another. This result may be correct only if each is a constant.

$$\text{Therefore,} \quad m_1\left[\sqrt{1 - u_1^{\ 2} / c^2}\right] = m_2\left[\sqrt{1 - u_2^{\ 2} / c^2}\right] = m_0$$

where, m_0 is the rest mass of the body.

$$\text{Thus,} \quad m_1 = \frac{m_0}{\sqrt{1 - u_1^{\ 2} / c^2}} \tag{ix}$$

$$\text{and} \quad m_2 = \frac{m_0}{\sqrt{1 - u_1^{\ 2} / c^2}} \tag{x}$$

In view of Eqs. (ix) and (x), we conclude that if m_0 be the rest mass of the body then its mass m when it moves at speed v will appear as

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}} \tag{xi}$$

This is the relativistic formula for the variation of mass with velocity.