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LONDON EQUATION

- The Maxwell's equations of electromagnetic waves were not able to explain the 'zero resistance' and 'ideal diamagnetism' of superconductors.
- F. London and H. London in the year 1935 derived two new equations to explain the superconducting state of matter.
- Hence, the equation of motions for the superconducting electrons in the presence of applied electric field E is given by
- http://nptel.ac.in/courses/115101012/4

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$$m\frac{d\vec{v}}{dt} = \vec{F} = -e\vec{E} \qquad \text{(i)}$$

The current density $\vec{J}$ can be expressed as

$$\vec{J} = -ne\vec{v} \qquad \text{(ii)}$$

where n is the number of electrons per unit volume. By differentiating Eq. (ii) w.r.t. time, we get

$$\frac{d\vec{J}}{dt} = -ne\frac{d\vec{v}}{dt} = -ne\left[-\frac{e\vec{E}}{m}\right]$$

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$$\frac{d\vec{J}}{dt} = \frac{ne^2}{m}\vec{E} \qquad \text{(iii)}$$

Eq. (iii) is known as *first London equation*. According to London's theory, it was assumed that two types of the electrons, i.e. normal and superconducting electrons are present in the superconductors. The normal electrons don't respond to the electric field and only the superconducting electrons respond to the electric field. Now Maxwell's equation can be written as

$$\text{curl }\vec{E} = -\frac{d\vec{B}}{dt}$$

$$\text{or} \quad \vec{\nabla}\times\vec{E} = -\mu_0\frac{d\vec{H}}{dt} \qquad \text{(iv)}$$

as $\vec{B} = \mu_0\vec{H}$.

Taking curl of Eq. (iii), we get

$$\text{curl }\frac{d\vec{J}}{dt} = \frac{ne^2}{m}\text{curl }\vec{E}$$

or

$$\vec{\nabla}\times\frac{d\vec{J}}{dt} = \frac{ne^2}{m}(\vec{\nabla}\times\vec{E}) \qquad \text{(v)}$$

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By using Eqs. (iv) and (v), we get

$$\vec{\nabla}\times\frac{d\vec{J}}{dt}=-\frac{\mu_0 ne^2}{m}\frac{d\vec{H}}{dt} \qquad \text{(vi)}$$

By integrating Eq. (vi) w.r.t. time, we get

$$\text{curl }\vec{J}=-\frac{\mu_0 ne^2}{m}[\vec{H}-\vec{H}_0] \qquad \text{(vii)}$$

where H_0 is a constant of integration. As we know that Meissner effect exhibits complete absence of magnetic field inside the superconductor. Therefore, $\vec{H}_0$ must be zero.Then

$$\text{curl }\vec{J}=-\frac{\mu_0 ne^2}{m}\vec{H}$$

or

$$\text{curl }\vec{J}=-\frac{ne^2}{m}\vec{B} \qquad \text{(viii)}$$

Eq. (viii) is known as *second London equation* which explains Meissner effect as well.