## Lecture06

## FIBRE <br> OPTICS

## Unit-03

## FIBRE OPTICS

## CONTENTS

Acceptance Angle
Numerical Aperture

## FIBRE OPTICS

## ACCEPTANCE ANGLE AND NUMERICAL APERTURE

- In order to propagate or transmit the light wave through the optical fibre, it is necessary to launch the light at angles that fall within certain range.
- The maximum limit of this angle is decided by the acceptance angle.


## Acceptance Angle

- Let us consider an optical fibre into which the light is incident.
- In Fig. we show a section of cylindrical optical fibre.
- The refractive index of the core is $\mu 1$ and that of the cladding is $\mu 2$ such that $\mu 1>$ $\mu 2$.


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- A light wave enters the fibre at an angle $\theta \mathrm{i}$ with the axis of the fibre. This wave gets refracted at an angle $\theta \mathrm{r}$ and strikes core-cladding interface at an angle $\theta$. If $\theta$ is greater than the critical angle $\theta \mathrm{c}$, the wave undergoes total internal reflection at the interface, since $\mu 1>\mu 2$.
- As long as the angle $\theta$ is greater than $\theta \mathrm{c}$, the light will stay within the core of the fibre.


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- Let us now compute the incident angle $\theta i$ for which $\theta i \geq \theta c$ such that the light refocusses within the core of the fibre. Applying Snell's law to the launching face of the fibre, we get

$$
\begin{equation*}
\frac{\sin \theta_{i}}{\sin \theta_{r}}=\frac{\mu_{1}}{\mu_{0}} \tag{i}
\end{equation*}
$$

If $\theta_{i}$ is increased beyond the limit, $\theta$ will drop below the critical value $\theta_{c}$ (as $\theta_{r}+\theta=90^{\circ}$, in $\triangle \mathrm{ABC}$ ) and the ray escapes from the side walls of the fibre. The largest value of $\theta_{i}$ occurs when $\theta=\theta_{c}$. This value of $\theta_{i}$ we represent by $\theta_{\text {imax }}$ From the $\triangle A B C$, it is seen that

$$
\begin{equation*}
\sin \theta_{r}=\sin \left(90^{\circ}-\theta\right)=\cos \theta,\left(\text { as } \theta_{r}+\theta=90^{\circ}\right) \tag{ii}
\end{equation*}
$$

From Eqs. (i) and (iii), we get

$$
\begin{equation*}
\frac{\sin \theta_{i}}{\cos \theta}=\frac{\mu_{1}}{\mu_{0}} \text { or } \sin \theta_{i}=\frac{\mu_{1}}{\mu_{0}} \cos \theta \tag{iii}
\end{equation*}
$$

when $\theta=\theta_{c}, \quad \sin \theta_{i}=\frac{\mu_{1}}{\mu_{0}} \cos \theta_{c}$
At critical angle, $\sin \theta_{c}=\frac{\boldsymbol{\mu}_{2}}{\boldsymbol{\mu}_{1}} \quad\left(\right.$ as $\left.\theta=90^{\circ}\right)$
$\therefore \quad \cos \theta_{c}=\sqrt{1-\sin ^{2} \theta_{c}}$

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$$
\begin{align*}
& =\sqrt{1-\frac{\mu_{2}^{2}}{\mu_{1}^{2}}} \\
\cos \theta_{c} & =\sqrt{\frac{\mu_{1}^{2}-\mu_{2}^{2}}{\mu_{1}^{2}}} \tag{iv}
\end{align*}
$$

or

By putting the value of $\cos \theta$ from Eq. (iv) into Eq. (iii), we get

$$
\begin{equation*}
\sin \theta_{i \max }=\frac{\sqrt{\mu_{1}^{2}-\mu_{2}^{2}}}{\mu_{0}} \tag{v}
\end{equation*}
$$

If the incident wave of light is launched from air medium (for which $\mu_{0}=1$ ), then putting $\theta_{\text {imax }}=\theta_{0}$, Eq. (v) may be simplified to

$$
\begin{equation*}
\sin \theta_{0}=\sqrt{\mu_{1}^{2}-\mu_{2}^{2}} \text { or } \theta_{0}=\sin ^{-1}\left(\sqrt{\mu_{1}^{2}-\mu_{2}^{2}}\right) \tag{vi}
\end{equation*}
$$

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## Numerical Aperture

- The angle $\theta 0$ is called the acceptance angle of the fibre, which may be defined as the maximum angle that a light wave can have relative to the axis of the fibre for its propagation through the fibre.
- The light wave contained within the cone having a full angle $2 \theta 0$ are accepted and transmitted along the fibre. Therefore, the cone associated with the angle $2 \theta 0$ is called the acceptance cone .
- The light incident at an angle beyond $\theta 0$ refracts through the cladding. As at every internal reflection the light will be lost being incident at an angle less than the critical angle, the corresponding optical energy is lost.
- It is also obvious that the acceptance angle would be larger if the diameter of the cone is larger.


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## Numerical Aperture

Numerical aperture (NA) is the most important parameter of an optical fibre. It is a measure of how much light can be collected by an optical system such as an optical fibre or a microscope lens. Based on the refractive indices of core and cladding, we can measure the value of NA. It is defined as the sine of the acceptance angle if the end faces of the fibre are exposed to a medium for which $\mu_{0}=1$ (air). Otherwise, the numerical aperture is defined as $N A=\mu_{0} \sin \theta_{0}$.
For $\mu_{0}=1, \quad N A=\sin \theta_{0}=\sqrt{\mu_{i}^{2}-\mu_{2}^{2}}$
This relation shows that the light gathering ability of an optical fibre increases with its numerical aperture. Since the maximum value of $\sin \theta_{0}$ can be 1 only, the value of $N A$ cannot exceed 1 . It means the largest value
of NA is unity. When $\theta_{0} \approx 90^{\circ}$, the fibre totally reflects all the light entering its face. Fibres with a wide variety of numerical apertures running from about 0.2 up to 1.0 and including 1.0 may commercially be obtained.

