Lecture-14

DIFFRACTION

Unit-01

DIFFRACTION

- RESOLVING POWER OF AN OPTICAL INSTRUMENT
- Rayleigh Criterion for Resolution
- RESOLVING POWER OF A PLANE DIFFRACTION GRATING
- DISPERSIVE POWER OF A PLANE DIFFRACTION GRATING

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RESOLVING POWER OF AN OPTICAL INSTRUMENT

- If two images are very close to each other, they may appear as one and it is impossible for the eye to see them separately.
- efore, some optical instruments are used to resolve such images. Our eye can resolve two objects only when the angle subtended by them at the eye is greater than one minute (1/60)°.
- Here we say that the resolving limit of the normal eye is (1/60)°.
- The resolving power of an optical instrument is defined as its ability to just resolve the images of two close point sources or small object.
- It is the ability of instrument to measure the angular separation of two images that are close to each other.

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Rayleigh Criterion for Resolution

- According to Rayleigh, two close images or two close spectral lines of equal intensities are said to be resolved by an optical instrument if the position of the central maxima of one spectral line coincides with the first minima of the other spectral line and vice-versa.
- The intensity distribution curves (diffraction pattern) of spectral lines having wavelengths λ and $\lambda + d\lambda$ of equal intensities are shown in Fig.

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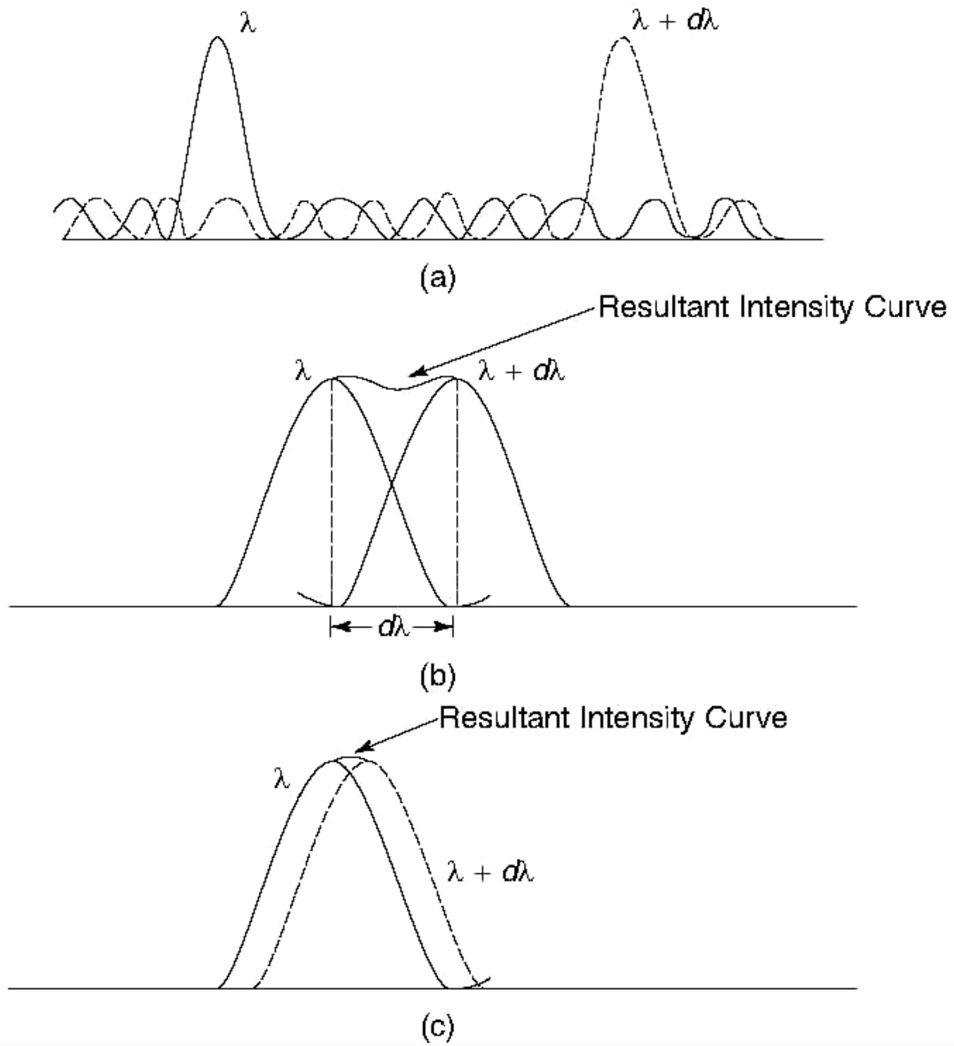

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- Fig. (a) says that when the difference in the angle of diffraction is large, the two spectral lines can be seen as separate ones and hence these spectral lines will be well resolved.
- In Fig. (b) say the difference in the angle of diffraction is such that the principal maxima of one just coincides with the first minima of the other. Here resultant intensity curve shows a dip in the middle of the central maxima of these spectral lines.
- According to Rayleigh, these spectral lines can be distinguished from one another and are said to be just resolved.

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- If the central maxima of two spectral lines corresponding to the wavelengths λ and $\lambda + d\lambda$ *are very close to each other, as shown in Fig.(c), then these two spectral lines overlap and they cannot be seen as separate ones.*
- According to Rayleigh criterion, two images or two close spectral lines of equal intensities are said to be just resolved when the resultant intensity at the dip is $(8/\pi 2)$ of the intensity of either central maxima. This can be proved as follows:

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According to the theory of single slit Fraunhofer diffraction,

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

Since first minima is formed at an angle $\alpha = \pi$, the angle at the point of intersection will be $\pi/2$. In this case, intensity of each curve at the dip

$$I_1 = I_2 = I_0 \frac{\sin^2(\pi/2)}{(\pi/2)^2} = \frac{4}{\pi^2} I_0$$

The resultant intensity at the dip

$$I = I_1 + I_2 = \frac{4}{\pi^2} I_0 + \frac{4}{\pi^2} I_0$$

$$= \frac{8}{\pi^2} I_0$$

$$= 0.81\, I_0$$

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RESOLVING POWER OF A PLANE DIFFRACTION GRATING

- Consider a parallel beam of light of wavelengths λ and λ + dλ incident normally on the plane transmission grating having grating element (b + d) and total number of rulings N.
- Then the resolving power of the grating is defined as the ratio of wavelength (λ) to the difference dλ of the wavelength. i.e., λ/ dλ.
- The separate diffraction pattern for λ and λ + dλ is shown in the Fig.
- According to Rayleigh criterion, these spectral lines are just resolved as the principal maxima of one line lies just on the first minima of the other.

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Now the direction of n^{th} principal maximum for a wavelength λ is given as

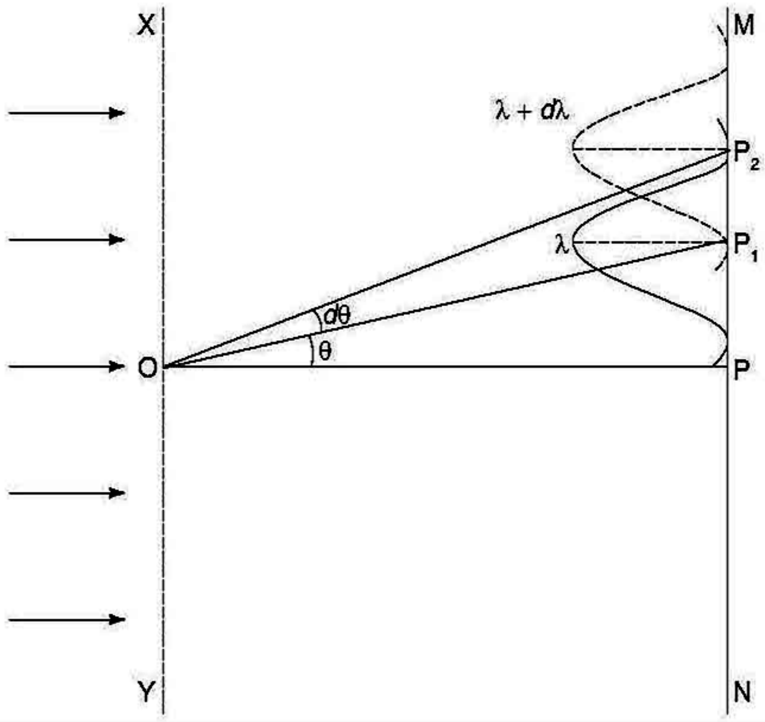
$$(b + d) \sin \theta = n\lambda \qquad \text{(i)}$$

The direction of n^{th} principal maximum for a wavelength $\lambda + d\lambda$ is given by

$$(b + d) \sin (\theta + d\theta) = n(\lambda + d\lambda) \qquad \text{(ii)}$$

The equation of minima for wavelength λ is

$$N(b + d) \sin \theta = m\lambda \qquad \text{(iii)}$$



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Here m has all the integral values except 0, N, 2N, ... nN, because for these values of m the condition for maxima is satisfied. Thus, first minimum adjacent to n^{th} principal maximum in the direction $(\theta + d\theta)$ can be obtained by substituting the value of m as $(nN + 1)$ in Eq. (iii). Therefore, first minima in the direction $(\theta + d\theta)$ is given by

$$N(b + d)\sin(\theta + d\theta) = (nN + 1)\lambda$$

$$\text{or}\quad (b + d)\sin(\theta + d\theta) = \frac{(nN + 1)\lambda}{N}$$

$$(b + d)\sin(\theta + d\theta) = n\lambda + \frac{\lambda}{N} \qquad \text{(iv)}$$

A comparison of Eq. (iv) with Eq. (ii), i.e. the Rayleigh criterion for just resolution, gives

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$\text{or}\quad n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$\text{or}\quad nd\lambda = \frac{\lambda}{N}$$

$$\text{or}\quad \frac{\lambda}{d\lambda} = nN$$

This is the required expression for the resolving power of the plane diffraction grating. This says that the number of lines per cm of a grating should be larger in order to increase its resolving power.

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DISPERSIVE POWER OF A PLANE DIFFRACTION GRATING

As we have seen, light of different wavelengths get dispersed/diffracted by the grating at different angles. In view of this, the angular dispersive power of a diffraction grating is defined as the rate of change of the angle of diffraction with the wavelength of light. It is denoted by $\frac{d\theta}{d\lambda}$.

For a plane transmission grating, the condition for principal maxima is

$$(b+d)\sin\theta = n\lambda \qquad \text{(i)}$$

where $(b + d)$ is the grating element and θ is the angle of diffraction for n^{th} order principal maxima. Differentiation of Eq. (i), w.r.t. λ gives

$$(b+d)\cos\theta\,\frac{d\theta}{d\lambda} = n$$

$$\therefore \qquad \Rightarrow \text{Dispersive power} = \frac{d\theta}{d\lambda} = \frac{n}{(b+d)\cos\theta} \qquad \text{(ii)}$$

Here $d\theta$ is the angular separation between the two lines having difference $d\lambda$ in the wavelengths.

We can make following observations from Eq. (ii).

(i) The dispersive power is directly proportional to n, i.e. to the order of diffraction. Thus, higher is the order, greater is the dispersive power. Hence the angular separation of two spectral lines is double in the second order diffraction as compared to that in the first order.

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(ii) The dispersive power is inversely proportional to the grating element $(b + d)$. This means that the dispersive power is directly proportional to number of lines per cm of grating. Therefore, the angular dispersive power of two given lines is greater with a grating having larger number of lines per cm.

(iii) The dispersive power is inversely proportional to $\cos\theta$. Thus, if the angle of diffraction $\theta = 0°$, $\cos\theta = 1$ and hence the angular dispersion is minimum. Therefore, if θ is small, the value of $\cos\theta$ may be taken as unity so the influence of $\cos\theta$ may be neglected.

If we neglect the influence of the factor $\cos\theta$, then $d\theta \propto d\lambda$ for a given order, i.e. the angular dispersion of two spectral lines in a particular order is directly proportional to the difference in the wavelengths. Such a spectrum is called a *normal spectrum*.

Linear Dispersive Power

If dx be the linear separation of two spectral lines differing in wavelengths by $d\lambda$ in the focal plane of a lens of focal length f, then we have

$$dx = f\,d\theta$$

Here, the linear dispersive power is defined as $\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda}$

$$= \frac{fn}{(b+d)\cos\theta} \qquad \text{(iii)}$$