Lecture-13

DIFFRACTION

Unit-01

DIFFRACTION

- SECONDARY MAXIMA
- Diffraction Pattern
- Missing Orders in Diffraction Pattern

DIFFRACTION

SECONDARY MAXIMA

- As discussed there are (N - 1) minima between two successive principal maxima. In order to differentiate two consecutive minima there should be a maximum between them. Therefore, there would be (N - 2) maxima between (N - 1) minima.
- These maxima are known as secondary maxima. The positions of secondary maxima are obtained by differentiating equation (iv) w.r.t. β and putting it to zero.

$$\frac{dI}{d\beta} = \frac{A_0^2 \sin^2\alpha}{\alpha^2} 2\left[\frac{\sin N\beta}{\sin\beta}\right]\left[\frac{N\cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0$$

or $N\cos N\beta \sin\beta - \sin N\beta \cos\beta = 0$, as $\sin\beta = 0$ gives principal maxima.

or $\tan N\beta = N \tan\beta$ (viii)

From this relation, we can draw the following triangle.

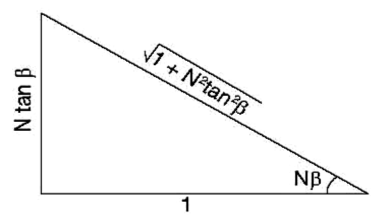

Fig. 2.17

From Fig. 2.17, we have

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1 + N^2\tan^2\beta}}$$

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Hence,
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta/(\sqrt{1+N^2\tan^2\beta})^2}{\sin^2\beta}$$

$$= \frac{N^2}{\cos^2\beta(1+N^2\tan^2\beta)}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta}$$

$$= \frac{N^2}{1-\sin^2\beta + N^2\sin^2\beta}$$

or
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta} \quad \text{(ix)}$$

Diffraction Pattern

As mentioned earlier $\frac{\sin^2\alpha}{\alpha^2}$ is the diffraction factor and $\sin^2 N\beta/\sin^2\beta$ is the interference factor. In Fig. 2.18 we plot these two separately and also a combined effect (product) of them is shown. Thus the intensity distribution or the diffraction pattern due to N slits or diffraction grating is shown in Fig. 2.18c.

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Form Eqs. (iv) and (ix), the intensity of secondary maxima is given by

$$I_s = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot \frac{N^2}{1+(N^2-1)\sin^2\beta} \qquad \text{(x)}$$

It is clear from Eq. (x) that the intensity of secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)\sin^2\beta}$. Since, the intensity of principal maxima is proportional to N^2.

$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

Hence, as N increases the intensity of secondary maxima decreases. In case of diffraction grating N is very large. Therefore, the secondary maxima are not visible in the spectrum and there is complete darkness between two successive principal maxima.

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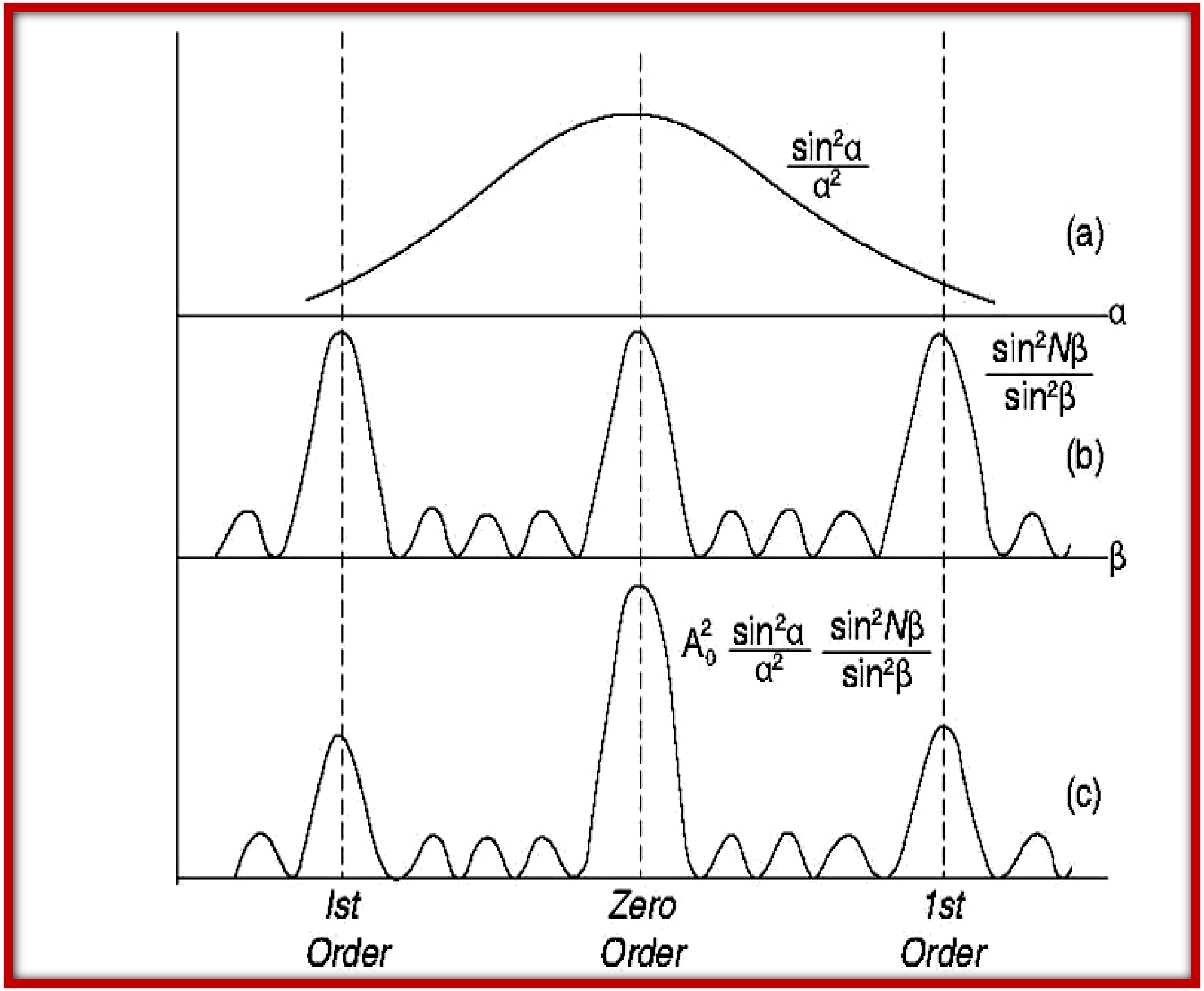

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Missing Orders in Diffraction Pattern

As observed in double slit diffraction, for a particular set of values of slit width and slit separation, certain number of interference maxima are found to be missing. In the case of diffraction grating also, similar situation arises for certain values of b and d. It means the condition of missing order is met in the diffraction pattern of diffractions grating. For this the conditions of interference maxima and diffraction minima should be simultaneously satisfied. The conditions of interference maxima is given by equation (vi). For the diffraction minima, it can be noted from diffraction factor $\sin^2\alpha/\alpha^2$ that it should be zero. It means $\sin\alpha = 0$ but $\alpha \neq 0$.

Therefore, $\mathbf{\sin\left(\frac{\pi}{\lambda} b \sin\theta\right) = 0}$. This would be true if

$$\frac{\pi}{\lambda} b \sin\theta = \pm m\pi$$

or $b\sin\theta = \pm m\lambda$.

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Therefore, in order to meet the condition of missing orders following relations should be satisfied.

$$(b + d)\sin\theta = n\lambda,\ \ n = 0, 1, 2, \ldots \qquad \text{(Interference maxima) (xi)}$$

$$b\sin\theta = m\lambda,\ m = 1, 2, 3, \ldots \qquad \text{(Diffraction minima) (xii)}$$

From Eqs. (xi) and (xii), we get

$$\frac{b+d}{b} = \frac{n}{m}$$

$$\text{or } n = \left(\frac{b+d}{b}\right)m$$

This is the condition of missing order of interference maxima in diffraction pattern. The absent orders are given below.

If $d = b$, then

$$n = \left(\frac{b+d}{b}\right)m = 2m$$

Therefore, for $m = 1, 2, 3, \ldots$ the 2^{nd}, 4^{th}, 6^{th}, … order interference will be absent as $n = 2, 4, 6\ldots$ If $d = 2b$, then, $n = 3, 6, 9, \ldots$ for $m = 1, 2, 3, \ldots$ Therefore the 3^{rd}, 6^{th}, 9^{th}, … order interference will be absent from the diffraction pattern.