Lecture-12

DIFFRACTION

Unit-01

DIFFRACTION

FRAUNHOFER DIFFRACTION BY N SLITS: DIFFRACTION GRATING

Analysis of Factor Sin2**nβ/Sin**2**β** Showing Interference Principal Maxima

DIFFRACTION

FRAUNHOFER DIFFRACTION BY N SLITS: DIFFRACTION GRATING

- We have seen that the intensity produced by double slits is four times of that of single slit. Therefore, it is expected that the device having more number of slits will produce large intensity
- So now we consider a large number of parallel slits of equal widths separated by equal opaque spaces. Such a device that makes use of multiple slit diffractions is called the diffraction grating.
- It is constructed by ruling large number of fine, equidistant and parallel lines on a optically plane glass plate with the help of fine diamond point.
- The ruled lines are opaque to light while the space between any two lines is transparent to the light and acts as a slit. There are about 15,000 lines per inch in such a grating.

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- Let a parallel, collimated beam of monochromatic light of wavelength λ be incident normally on N-parallel slits (grating) each of width b and separated by a opaque distance d. The sum of b and d is known as grating element.
- The middle points in two consecutive slits separated by the distance (b + d) are known as corresponding points. Let the diffracted light be focused by a convex lens L on the screen XY placed in the focal plane of the lens. All the secondary waves traveling in the direction parallel to the direction of incidence are brought to focus at a point C .
- The point C corresponds to the position of central bright maximum. The rays making an angle θ with the direction of incidence are focused at a point P.

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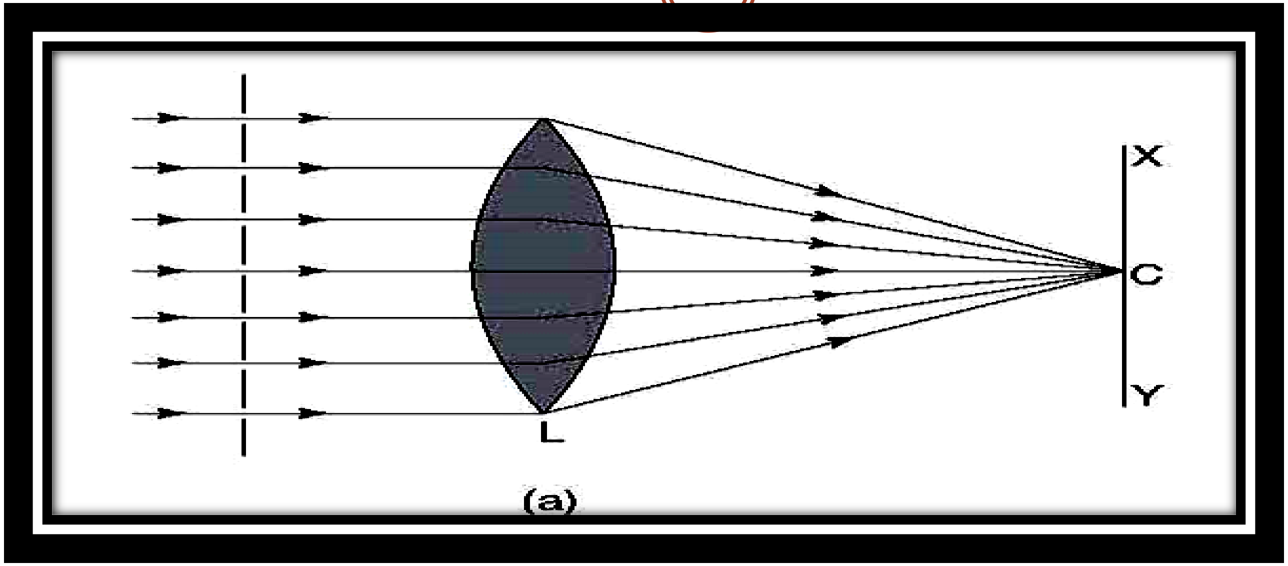

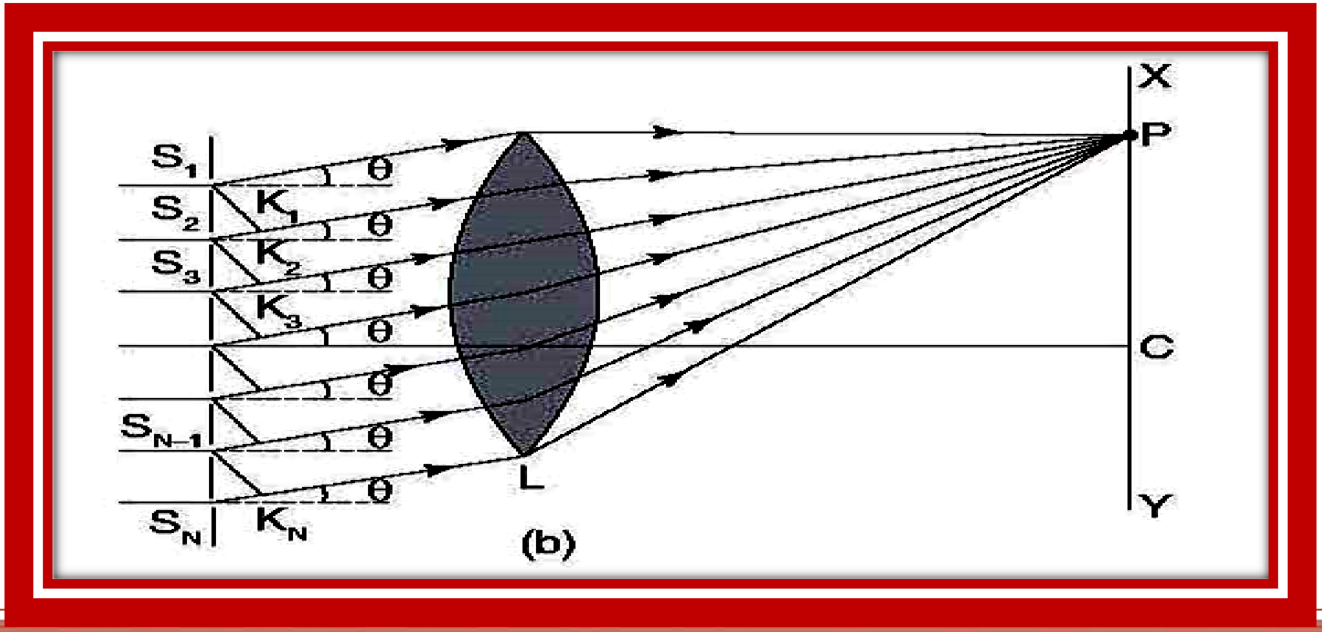

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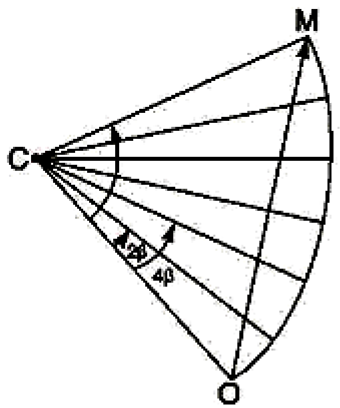
We may consider that the each slit in the grating is equivalent to an individual coherent source which is placed at the middle of each slit and sending a single wave of amplitude $\frac{A\sin\alpha}{\alpha}$ at angle θ with the direction of wave propagation. Here $\alpha = \frac{\pi}{\lambda} \times b\sin\theta$.

If S_1K_1 be the perpendicular on S_2K_1, then the path difference between the waves originating from S_1 and S_2 is given by

$$S_2K_1 = (b + d)\sin\theta \qquad \text{(i)}$$

The corresponding phase difference would be

$$\frac{2\pi}{\lambda}(b + d)\sin\theta = 2\beta \text{ (say)} \qquad \text{(ii)}$$



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It is clear from the above equation that the phase difference between two successive waves is constant and is equal to 2β. The phases increase in arithmetical progression (Fig. 2.16). Thus, we can construct the polygon of N amplitudes. The resultant amplitude and intensity at point P due to the waves from N slits can be obtained by vector polygon method and is given by

$$R = R_0 \frac{\sin N\beta}{\sin\beta} = \frac{A_0 \sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta} \qquad \text{(iii)}$$

A measure of the resultant intensity at point P is given by

$$I = R^2 = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta} \qquad \text{(iv)}$$

The factor $\frac{A_0^2 \sin^2\alpha}{\alpha^2}$ gives the intensity distribution in diffraction pattern due to a single slit, while the factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ yields the interference pattern due to N-slits.

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Analysis of Factor Sin2nβ/Sin2β Showing Interference Principal Maxima

Now we discuss principal maxima, minima and seconadry maxima obtained by diffraction grating.

When $\sin\beta = 0$ or $\beta = \pm n\pi$, where $n = 0, 1, 2, \ldots$ etc. and $\sin N\beta = 0$, we get $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$. It means it is indeterminate. Therefore, in order to evaluate the value of $\frac{\sin N\beta}{\sin\beta}$ we differentiate the numerator and denominator according to L′ Hospital rule. Thus,

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}\sin\beta}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

By substituting this value of $\frac{\sin N\beta}{\sin\beta}$ in Eq. (iv), we have

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} N^2 \qquad \text{(v)}$$

That is the resultant intensity of maxima becomes $\frac{A_0^2 \sin^2\alpha}{\alpha^2} N^2$. Therefore, the resultant intensity of any of the principal maxima in the diffraction pattern can be obtained by multiplying N^2 to the factor $\frac{A_0^2 \sin^2\alpha}{\alpha^2}$. Being proportional to N^2, the brightness of the principal maxima increases with the increase of number of slits. These maxima are obtained in the direction given by

$$\beta = \pm n\pi$$

$$\text{or } \frac{\pi}{\lambda}(b + d)\sin\theta = \pm n\pi$$

$$\text{or } (b + d) \sin\theta = \pm n\lambda \qquad \text{(vi)}$$

where $n = 0, 1, 2, \ldots$

For $n = 0$, we get $\theta = 0$ which gives zero order principal maximum. For the other values of n as 1, 2, 3, … we obtain first, second, third, … order principal maximum, respectively. The condition for the existence of a principal maximum is sometimes called the diffraction grating equation. The value of n gives the order the diffraction.

Minima

The intensity expression (iv) shows that it is minimum when $\sin N\beta = 0$ but $\sin\beta \neq 0$.

Therefore, $\sin N\beta = 0$ or $N\beta = \pm m\pi$

$$\text{or } N(\pi/\lambda)(b + d) \sin\theta = \pm m\pi$$

$$\text{or } N(b + d) \sin\theta = \pm m\lambda \qquad \text{(vii)}$$

where m can have all integral values except 0, N, 2N, 3N …, nN. This is because $\sin\beta = 0$ for these values of m, which gives different principal maxima.

It is clear from Eq. (vii) that $m = 0$ gives principal maximum of zero order. $m = 1, 2, 3, \ldots (N - 1)$ give minima and $m = N$ gives again principal maximum of first order. Thus, there are $(N - 1)$ equispaced minima between two consecutive principal maxima.