

Lecture-  
09

# DIFFRACTION

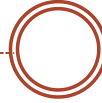
Unit-01

# DIFFRACTION

FRAUNHOFER DIFFRACTION BY A SINGLE SLIT

Conditions of Maxima and Minima

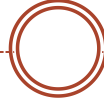
# DIFFRACTION



## FRAUNHOFER DIFFRACTION BY A SINGLE SLIT

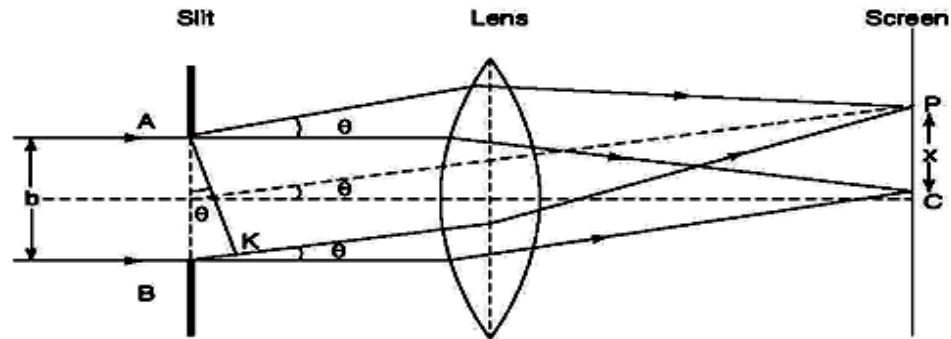
- Since in Fraunhofer diffraction the source is effectively at infinite distance, a collimated parallel beam of monochromatic light of wavelength  $\lambda$  can be taken as incident normally on a narrow slit AB of width  $b$ .
- We divide this wavefront into a large number of points  $n$  each sending waves of equal amplitude according to Huygens' principle. These waves get diffracted and then interfere to produce diffraction pattern on the screen.
- The secondary waves that travel along the direction of incident beam are focused at point C while those inclined at an angle  $\theta$  with the direction of incident beam (due to diffraction) are focused at another point P.
- To find out the resultant intensity at P, we draw a perpendicular AK on BK. It is clear from the figure that the optical paths of the waves traveled after the plane AK to the point P are equal. However, the optical paths of the waves originating from points on AB (from A toward B) and reaching the point P gradually increase.

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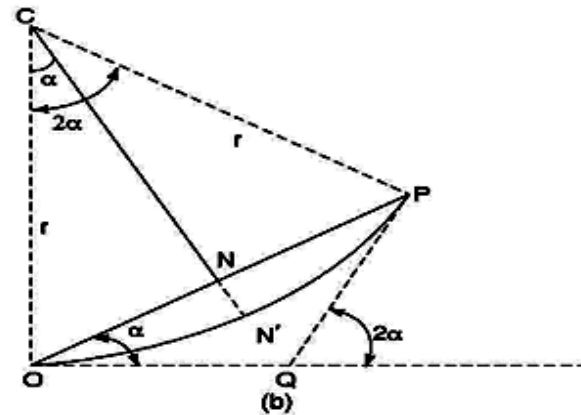
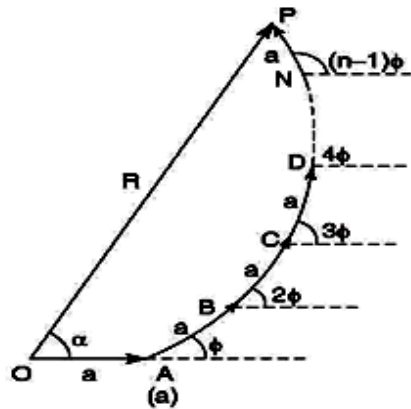


- Hence, the phase difference between them gets larger. This is shown in Fig. for  $n$  number of waves each of amplitude  $a$ .
- **In this figure a phase difference of  $\phi$  is taken between two successive waves. The resultant amplitude of these waves at point P is shown as R.**
- The total path difference between the waves originating from extreme points A and B is  $BK = AB \sin\theta = b \sin\theta$ .
- Therefore, the path difference between different waves originating from all the points of the slit AB vary between zero and  $b \sin\theta$ .
- The phase difference corresponding to path difference  $b \sin\theta$  will be  $(2\pi/\lambda) b \sin\theta$

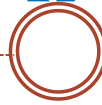
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Since the aperture is divided into  $n$  equal parts, the phase difference between any two consecutive parts will be  $\frac{1}{n} \frac{2\pi}{\lambda} b \sin \theta$  ( $= \phi$ , say).



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- The resultant amplitude and intensity at point P due to all these secondary waves can be obtained by vector polygon method.
- Let  $\alpha$  be the phase difference between the waves from the initial direction to the resultant, then  $2\alpha$  will be the total phase difference between the secondary waves originating from extreme points of the slit AB.
- Here, it is taken that all the amplitudes constitute an arc due to their large number and small phase difference between them. Because of the symmetry, we have  $\angle O = \alpha$  and  $\angle Q = 2\alpha$ .
- The chord OP gives the resultant amplitude due to all the secondary waves at point P.

$$\sin \alpha = \frac{ON}{OC} = \frac{ON}{r}$$

$$\text{or } ON = r \sin \alpha$$

(i)

where,  $r$  is the radius of the circular arc.

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$$\therefore \text{Chord } OP = 2ON = 2r \sin \alpha$$

$$\therefore \text{Chord } OP = \text{resultant amplitude}$$

$$\therefore R = 2r \sin \alpha$$

The length of the arc  $ON'P = na$ , where  $n$  is an integer number and  $a$  is the amplitude of each vibration (Fig. 2.9 b).

We know that,

$$\angle PCO = 2\alpha = \frac{\text{Arc } ON'P}{\text{Radius}} = \frac{na}{r}$$

$$\text{or } 2r = \frac{na}{\alpha} \quad (\text{iii})$$

Substituting the value of  $2r$  in Eq. (ii), we get

$$R = na \frac{\sin \alpha}{\alpha}$$

$$R = A_0 \frac{\sin \alpha}{\alpha} \quad (\text{iv})$$

Here we have taken  $na = A_0$ . Thus, resultant intensity at point P on the screen is given by a measure of

$$I = R^2 = A_0^2 \frac{\sin^2 \alpha}{\alpha^2} = I_0 \left[ \frac{\sin \alpha}{\alpha} \right]^2$$

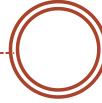
$$I = I_0 \left[ \frac{\sin \alpha}{\alpha} \right]^2 \quad (\text{v})$$

Thus, the magnitude of the resultant intensity at any point on the screen is a function of  $\alpha$  and the slit width  $b$ . Since the phase difference of  $2\alpha$  is introduced due to the path difference of  $b \sin \theta$ .

$$\therefore 2\alpha = \frac{2\pi}{\lambda} \times b \sin \theta$$

$$\text{or } \alpha = \frac{\pi}{\lambda} \times b \sin \theta$$

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## Conditions of Maxima and Minima

It is clear from Eq. (iv) that the resultant amplitude  $R$  will be a maximum when

$$\frac{\sin \alpha}{\alpha} = 1, \text{ which is true when } \alpha \rightarrow 0, \text{ i.e.,}$$

$$\frac{\pi}{\lambda} b \sin \theta \rightarrow 0 \text{ or } \sin \theta \rightarrow 0$$

$$\Rightarrow \theta = 0^\circ$$

From Eq. (iv),

$$R = \frac{A_0}{\alpha} \sin \alpha = \frac{A_0}{\alpha} \left[ \alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \frac{\alpha^7}{\lfloor 7} + \dots \right]$$

$$= A_0 \left[ 1 - \frac{\alpha^2}{\lfloor 3} + \frac{\alpha^4}{\lfloor 5} - \frac{\alpha^6}{\lfloor 7} + \dots \right]$$

$$R = A_0 \text{ for } \alpha = 0.$$

Hence the intensity corresponding to  $\alpha = 0$  is  $I = R^2 = A_0^2 = I_0$ . This is called as the intensity of central (principal) maximum.



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The first relation  $y = \alpha$  represents the equation of straight line passing through the origin making an angle  $45^\circ$  with the axis and the equation  $y = \tan \alpha$  represents a discontinuous curve having a number of branches with asymptotes at the intervals of  $\pi$  (Fig. 2.10). The points of intersections of these curves will give the values of  $\alpha$  that will satisfy the relation  $\alpha = \tan \alpha$ .

Therefore, the maxima occur when

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \dots \text{ or } \alpha = (2n + 1)\frac{\pi}{2}, \quad n = 1, 2, 3 \dots$$

These are called points of secondary maxima. A measure of intensity of first secondary maxima is obtained from Eq. (v) with  $\alpha = \frac{3\pi}{2}$ , as

$$I_1 = A_0^2 \left[ \frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right]^2 = \frac{4}{9\pi^2} I_0$$

Similarly, the intensity of second secondary maxima is

$$I_2 = A^2 \left[ \frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}} \right]^2 = \frac{4}{25\pi^2} I_0$$

Similarly,

$$I_3 = A^2 \left[ \frac{\sin\left(\frac{7\pi}{2}\right)}{\frac{7\pi}{2}} \right]^2 = \frac{4}{49\pi^2} I_0 \quad \text{and so on.}$$

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In order to determine the position of maximum intensity, let us differentiate Eq (v) w.r.t. to  $\alpha$  and equate it to zero, i.e.

$$\frac{dI}{d\alpha} = 2I_0 \frac{\sin \alpha}{\alpha} \left[ \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} \right] = 0$$

Since  $I_0$  cannot be zero, either  $\sin \alpha = 0$  or  $\alpha \cos \alpha - \sin \alpha = 0$ . The equation  $\sin \alpha = 0$  determines the positions of minima [Eq. (iv)] except when  $\alpha = 0$  because it corresponds to the position of maximum. Therefore, the following condition should be satisfied for obtaining the minima

$$\sin \alpha = 0 \quad \text{or} \quad \alpha = \pm m\pi$$

or 
$$\frac{\pi}{\lambda} b \sin \theta = \pm m\pi$$

or 
$$b \sin \theta = \pm m\lambda, m = 1, 2, 3\dots$$

The position of maxima are given by

$$\alpha \cos \alpha - \sin \alpha = 0$$

$$\text{or} \quad \alpha = \tan \alpha$$

This equation can be solved graphically by plotting the curves

$$y = \alpha$$

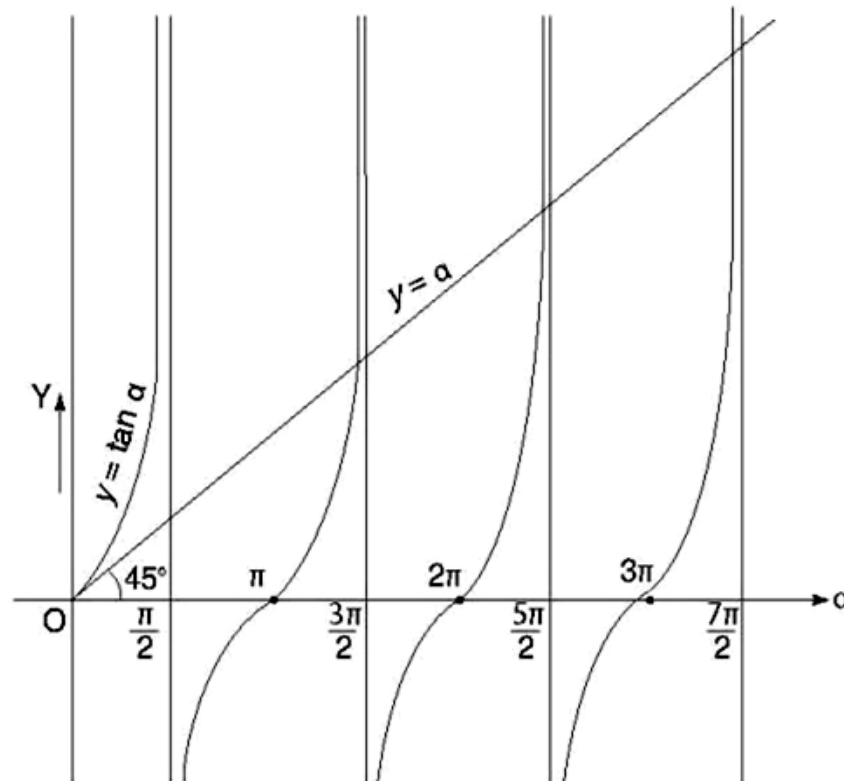
$$\text{and } y = \tan \alpha$$

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Thus, the ratio of relative intensities of successive maxima are

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \dots$$

The intensity of the first secondary maxima  $\frac{4}{9\pi^2}$ , i.e., 4.5% that of principal maximum as shown in Fig. 2.11.



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