Mathematics Question Bank

Calculus

Q. 1 The value of the function f(x) = $\lim_{x\to 0} \frac{x^3+x^2}{2x^3-7x^2}$ is

(a) 0 (b) $-\frac{1}{7}$

(c) $\frac{1}{7}$ (d) ∞

Q. 2 The $\lim\limits_{x\to 0} \frac{sin\frac{2x}{3}}{x}$ is

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{3}{2}$ (d) ∞

Q. 3 The expression $\lim\limits_{\alpha\to 0} \frac{x^{\alpha}-1}{\alpha}$ is equal to

(a) log x (b) 0

(c) xlogx (d) ∞

Q. 4 What is the value of $\lim\limits_{x\to 0} \frac{e^x-(1+x+\frac{x^2}{2})}{x^3}$

(a) 0 (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) 1

Q. 5 The value of $\lim\limits_{x\to 0} (\frac{1-cosx}{x^2})$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

Q. 6 The value of $\lim_{x\to 0} (\frac{-sinx}{2sinx+cosx})$ is

(a) 0 (b) 2

(c) 3 (d) 1

Q. 7 What is the value of $\lim_{n\to\infty} (1-\frac{1}{n})^{2n}$

(a) 0 (b) e^{-2}

(c) $e^{-1/2}$ (d) 1

Q. 8 The value of $\lim_{x\to\infty} (1+x^2)^{e^{-x}}$ is

(a) 0 (b) 1/2

(c) 1 (d) ∞

Q. 9 What should be the value of λ such that the function defined below is continuous at x = π/2 ? Given f(x) = $\begin{cases} \frac{\lambda cosx}{\frac{\pi}{2}-x} \ if \ x \neq \pi/2 \\ 1 \ if \ x = \pi/2 \end{cases}$

(a) 0 (b) 2/ π

(c) 1 (d) π/2

Q. 10 The function y = $|2-3x|$

(a) is continuous ∀ x ϵ R and differentiable ∀ x ϵ R.

(b) is continuous ∀ x ϵ R and differentiable ∀ x ϵ R except at x=3/2.

(c) is continuous ∀ x ϵ R and differentiable ∀ x ϵ R except at x=2/3.

(d) is continuous ∀ x ϵ R except x = 3 and differentiable ∀ x ϵ R.

Q. 11 Consider the function f(x) = $|x|$ in the interval -1 $< x \leq 1$.At the point x = 0, f(x) is

(a) continuous and differentiable
(b) non continuous and differentiable
(c) continuous and non differentiable
(d) neither continuous nor differentiable

Q. 12 Which of the following functions is continuous at x = 3?

I.f(x) = $\begin{cases} 2 \ if \ x = 3 \\ x - 1 \ if \ x > 3 \\ \frac{x+3}{3} \ if \ x < 3 \end{cases}$

II.f(x) = $\begin{cases} 4 \ if \ x = 3 \\ 8 - x \ if \ x \neq 3 \end{cases}$

III.f(x) = $\begin{cases} x + 3 \ if \ x \leq 3 \\ x - 4 \ if \ x > 3 \end{cases}$

IV.f(x) = $\frac{1}{x^3 - 27}$ if x $\neq$ 3

(a) I.
(b) II.
(c) III.
(d) IV.

Q. 13 Let the function f(θ) = $\begin{vmatrix} sin\theta & cos\theta & tan\theta \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) & \tan\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & \tan\left(\frac{\pi}{3}\right) \end{vmatrix}$ where $\theta \epsilon \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and f′ (θ) denote the derivative of f with respect to θ.Which of the following statements is/are true?

(I) There exists $\theta \epsilon$ ($\frac{\pi}{6}, \frac{\pi}{3}$) such that f′ (θ) = 0.

(II) There exists $\theta \epsilon$ ($\frac{\pi}{6}, \frac{\pi}{3}$) such that f′ (θ) ≠ 0.

(a) (I) only
(b) (II) only
(c) Both (I) and (II)
(d) Neither (I) nor (II)

Q. 14 The function f(x) = xsinx satisfies : f′ ′ (x) + f(x) + tcosx = 0. The value of t is

(a) -2
(b) 2

(c) 1 (d) -1

Q. 15 A rail engine accelerates from its stationary position for 8 seconds and travels a distance of 280 m. According to the mean value theorem, the speedometer at a certain time during acceleration must read exactly

(a) 0 (b) 8 kmph

(c) 75 kmph (d) 126 kmph

Q. 16 A function f(x) = $1 - x^2 + x^3$ is defined in the closed interval [-1 , 1]. The value of x in the open interval (-1 , 1) for which the mean value theorem is satisfied, is

(a) -1/2 (b) -1/3

(c) 1/3 (d) 1/2

Q. 17 If x = $a(\theta + sin\theta)$ and y = $a(1 - \cos\theta)$, then dy/dx will be equal to

(a) $\sin\left(\frac{\theta}{2}\right)$ (b) $\cos\left(\frac{\theta}{2}\right)$

(c) $\tan\left(\frac{\theta}{2}\right)$ (d) $\cot\left(\frac{\theta}{2}\right)$

Q. 18 As x increases from $-\infty \; to \; \infty$, the function f(x) = $\frac{e^x}{1+e^x}$

(a) monotonically increases (b) monotonically decreases

(c) increases to a maximum value and then decreases (d) decreases to a minimum value and then increases

Q. 19 Given a function f(x,y) = $4x^2 + 6y^2 - 8x - 4y + 8$. The optimal value of f(x,y) is

(a) a minimum equal to 10/3 (b) a maximum equal to 10/3

(c) a minimum equal to 8/3 (d) is a maximum equal to 8/3

Q. 20 While minimizing the function f(x), necessary and sufficient conditions for a point x_0 to be a minima are

(a) $f'(x_0) > 0$ and $f''(x_0) = 0$

(b) $f'(x_0) < 0$ and $f''(x_0) = 0$

(c) $f'(x_0) = 0$ and $f''(x_0) < 0$

(d) $f'(x_0) = 0$ and $f''(x_0) > 0$

Q. 21 The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is

(a) 1

(b) $\sqrt{3}/2$

(c) $\sqrt{3}$

(d) 2

Q. 22 At x = 0, the function f(x) = $|x|$ has

(a) A minimum

(b) A maximum

(c) A point of inflexion

(d) Neither a maximum nor minimum

Q. 23 The function $f(x) = 2x – x^2 + 3$ has

(a) A maxima at x = 1 and a minima at x = 5

(b) A maxima at x = 1 and a minima at x = -5

(c) Only a maxima at x =1

(d) Only a minima at x = 1

Q. 24 If the sum of the diagonal elements of a 2 x 2 symmetric matrix is -6, then the maximum possible value of determinant of the matrix is

(a) 9

(b) 8

(c) 25

(d) 10

Q. 25 For a right angled triangle, if the sum of the lengths of the hypotenuse and a side is kept constant, in order to have maximum area of the triangle,the angle between the hypotenuse and the side is

(a) 12^0 (b) 36^0

(c) 60^0 (d) 45^0

Q. 26 The maximum area (in square unit) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is

(a) 1 (b) 2

(c) 4 (d) 3

Q. 27 The function f(x) = $2x^3 - 3x^2 - 36x + 2$ has its maxima at

(a) x = -2 only (b) x = 0 only

(c) x = 3 only (d) both x = -2 and x = 3

Q. 28 The minimum value of function y = x^2 in the interval [1,5] is

(a) 0 (b) 1

(c) 25 (d) undefined

Q. 29 Consider function f(x) = $(x^2 - 4)^2$ where x is a real number. Then the function has

(a) only one minimum (b) only two minima

(c) three minima (d) three maxima

Q. 30 A function y = $5x^2 + 10x$ is defined over an open interval x = (1,2). At least at one point in this interval, dy/dx is exactly

(a) 20 (b) 25

(c) 30 (d) 36

Q. 31 Consider the function f(x) = sinx in the interval x ε [$\frac{\pi}{4}$, 7 $\frac{\pi}{4}$]. The number and the location(s) of the local minima of this function are

(a) One, at $\pi/2$
(b) One, at 3 $\pi/2$
(c) Two, at $\pi/2$ and 3 $\pi/2$
(d) Two, at $\pi/4$ and 3 $\pi/2$

Q. 32 The infinite series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ corresponds to

(a) secx
(b) e^x
(c) cosx
(d) 1 + $\sin^2$x

Q. 33 For the function e^{-x} , the linear approximation around x = 2 is

(a) (3 –x)e^{-2}
(b) 1 - x
(c) [3 + 2$\sqrt{2}$ – (1 + $\sqrt{2}$)x] e^{-2}
(d) e^{-2}

Q. 34 Which of the following functions would have only odd powers of x in its Taylor's series expansion about the point x = 0

(a) sin (x^3)
(b) sin (x^2)
(c) cos (x^3)
(d) cos (x^2)

Q. 35 In the Taylor's series expansion of exp(x) + sin(x) about the point x = π, the coefficient of $(x-\pi)^2$ is

(a) exp(π)
(b) 0.5 exp(π)
(c) exp(π) + 1
(d) exp(π) - 1

Q. 36 The Taylor's series expansion of $\frac{sinx}{x-\pi}$ at x = π is given by

(a) $1 + \frac{(x-\pi)^2}{3!} + \cdots$
(b) $-1 - \frac{(x-\pi)^2}{3!} + \cdots$

(c) $1 - \frac{(x-\pi)^2}{3!} + \cdots$

(d) $-1 + \frac{(x-\pi)^2}{3!} + \cdots$

Q. 37 Let f = y^x. What is $\frac{\partial^2 f}{\partial x \partial y}$ at x = 2, y = 1?

(a) 0

(b) log2

(c) 1

(d) 1/log2

Q. 38 Let z = xy log(xy), then

(a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

(b) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

(d) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

Q. 39 The integral $\int_0^\pi sin^3\theta d\theta$ is given by

(a) 1/2

(b) 2/3

(c) 4/3

(d) 8/3

Q. 40 The integral $\int_0^{\pi/4} \frac{1-tanx}{1+tanx} dx$ evaluates to

(a) 0

(b) 1

(c) log2

(d) $\frac{1}{2}$log2

Q. 41 Given i = $\sqrt{-1}$, what is the value of $\int_0^{\pi/2} \frac{cosx+isinx}{cosx-isinx} dx$?

(a) 0

(b) 2

(c) -i

(d) i

Q. 42 Evaluate $\int_0^\infty \frac{sint}{t} dt$

(a) π (b) $\pi/2$

(c) $\pi/4$ (d) $\pi/8$

Q. 43 Which of the following integrals is unbounded ?

I. $\int_0^{\pi/4} tanx\, dx$ II. $\int_0^{\infty} \frac{1}{x^2+1}\, dx$ III. $\int_0^{\infty} xe^{-x} dx$ IV. $\int_0^1 \frac{1}{1-x}$dx

(a) I. (b) II.

(c) III. (d) IV.

Q. 44 If for non-zero x, af(x) + bf($\frac{1}{x}$) = $\frac{1}{x}$ – 25 where a ≠ b then $\int_1^2 f(x)$dx is

(a) $\frac{1}{a^2-b^2}[a(log2 - 25) + \frac{47b}{2}]$ (b) $\frac{1}{a^2-b^2}[a(2log2 - 25) - \frac{47b}{2}]$

(c) $\frac{1}{a^2-b^2}[a(2log2 - 25) + \frac{47b}{2}]$ (d) $\frac{1}{a^2-b^2}[a(log2 - 25) - \frac{47b}{2}]$

Q. 45 What is the value of the definite integral, $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$?

(a) 0 (b) a/2

(c) a (d) 2a

Q. 46 The value of $\int_0^{\pi/6} cos^4 3\theta sin^3 6\theta d\theta$ is

(a) 0 (b) 1/15

(c) 1 (d) 8/3

Q. 47 The value of the integral $\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2+\cos(x-1)} dx$ is

(a) 3 (b) 0

(c) -1 (d) -2

Q. 48 If $\int_0^{2\pi} |x sinx| dx = k\pi,$,then the value of k is equal to

(a) 3 (b) 4

(c) 6 (d) 0

Q. 49 What is the area of the segment cut off from the parabola $x^2 = 8y$ by the line $x-2y + 8 = 0$

(a) 36 (b) 45

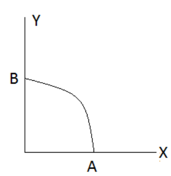
(c) 48 (d) 25

Q. 50 What is the area common to the circles r = a and r = $2a cos\theta$

(a) $0.524a^2$ (b) $0.614a^2$

(c) $1.047a^2$ (d) $1.228a^2$

Q. 51 A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x + y)^2$ on path AB traversed in a counter clockwise sense is



(a) $\frac{\pi}{2} - 1$ (b) $\frac{\pi}{2} + 1$

(c) $\frac{\pi}{2}$ (d) 1

Q. 52 The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(a) 16/3 (b) 8

(c) 32/3 (d) 16

Q. 53 The parabolic arc y = $\sqrt{x}$, 1 ≤ x ≤ 2 is revolved around the x-axis. The volume of the solid of revolution is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

Q. 54 The area enclosed between the straight line y = x and the parabola y = x^2 in the xy plane is

(a) 1/6 (b) 1/4

(c) 1/3 (d) 1/2

Q. 55 Consider an ant crawling along the curve $(x – 2)^2 + y^2 = 4$ where x and y are in meters. The ant starts at the point (4, 0) and moves counter clockwise with a speed 1.57 meters/second. The time taken by the ant to reach the point (2, 2) is (in seconds)

(a) 3 (b) 2

(c) 1 (d) 4

Q. 56 The value of the integral of the function g(x, y) = $4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (1, 2) in the x-y plane is

(a) 33 (b) 35

(c) 40 (d) 56

Q. 57 The volume of an object expressed in spherical co-ordinates is given by

V = $\int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 sin\emptyset dr d\emptyset d\theta$. The value of the integral is

(a) $\pi/3$ (b) $\pi/6$

(c) $2\pi/3$ (d) $\pi/4$

Q. 58 Changing the order of integration in the double integral I = $\int_0^8 \int_{x/4}^2 f(x,y)dydx$ leads to I = $\int_r^s \int_p^q f(x,y)dxdy$. What is q?

(a) 4y (b) $16y^2$

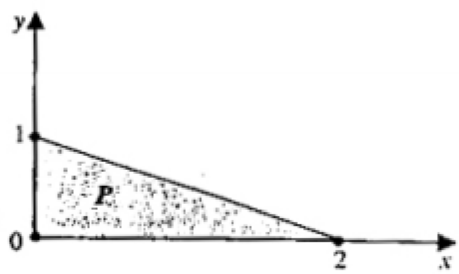
(c) x (d) 8

Q. 59 By a change of variable x(u, v) = uv, y(u, v) = v/u in a double integral, the integrand f(x, y) changes to f(uv, v/u)Ø(u, v). Then Ø(u, v) is

(a) 2v/u (b) 2uv

(c) v^2 (d) 1

Q. 60 Consider the shaded triangular region P shown in the figure. What is $\iint_P xydxdy$?



(a) 1/6 (b) 2/9

(c) 7/16 (d) 1

Q. 61 The value of the integral $\int_0^2 \int_0^x e^{x+y} dydx$

(a) $\frac{1}{2}(e-1)$ (b) $\frac{1}{2}(e^2-1)^2$

(c) $\frac{1}{2}(e^2-e)$ (d) $\frac{1}{2}(e-\frac{1}{e})^2$

Q. 62 To evaluate the double integral $\int_0^8 (\int_{\frac{y}{2}}^{(\frac{y}{2})+1} (\frac{2x-y}{2})dx)dy$, we make the substitution u =

$(\frac{2x-y}{2})$ and v = $\frac{y}{2}$. The integral will reduce to

(a) $\int_0^4(\int_0^2 2udu)dv$

(b) $\int_0^4(\int_0^1 2udu)dv$

(c) $\int_0^4(\int_0^1 udu)dv$

(d) $\int_0^4(\int_0^2 udu)dv$

Q. 63 The volume enclosed by the surface f(x, y) = e^x over the triangle bounded by the lines x = y; x = 0, y = 1 in the xy plane is

(a) e - 1

(b) e - 2

(c) e

(d) e + 2

Q. 64 The double integral $\int_0^a \int_0^y f(x,y)dx\,dy$ is equivalent to

(a) $\int_0^x \int_0^y f(x,y)dx\,dy$

(b) $\int_0^a \int_x^y f(x,y)dx\,dy$

(c) $\int_0^a \int_x^a f(x,y)dy\,dx$

(d) $\int_0^a \int_0^a f(x,y)dx\,dy$

Q. 65 Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z}(x+y+z)dxdydz$

(a) 1

(b) 4

(c) 0

(d) 9

Q. 66 Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$

(a) 1/48

(b) 48

(c) 1/24

(d) 0

Q. 67 If P, Q , R are three points having co-ordinates (3 , -2 , -1) , (1 , 3 , 4) , (2 , 1 , -2) in XYZ space, then the distance from the point P to plane OQR , (O being the origin of the co-ordinate system) is given by

(a) 3

(b) 5

(c) 7 (d) 9

Q. 68 The directional derivative of f(x,y,z) = $2x^2 + 3y^2 + z^2$ at the point P(2, 1, 3) in the direction of the vector a = i – 2k is

(a) -2.785 (b) -2.145

(c) -1.789 (d) 1.000

Q. 69 A velocity vector is given as $\vec{V} = 5xy\vec{i} + 2y^2\vec{j} + 3yz^2\vec{k}$. The divergence of this velocity vector at (1, 1, 1) is

(a) 9 (b) 10

(c) 14 (d) 15

Q. 70 For a scalar function f(x, y, z) = $x^2 + 3y^2 + 2z^2$, the gradient at the point P(1, 2, -1) is

(a) $2\vec{i} + 6\vec{j} + 4\vec{k}$ (b) $2\vec{i} + 12\vec{j} - 4\vec{k}$

(c) $2\vec{i} + 12\vec{j} + 4\vec{k}$ (d) $\sqrt{56}$

Q. 71 For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ is given by

(a) $\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ (b) $\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$

(c) $\vec{k}$ (d) $\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$

Q. 72 Curl of a vector V(x, y, z) = $2x^2\vec{i} + 3z^2\vec{j} + y^3\vec{k}$ at x = y = z = 1 is

(a) $-3\vec{i}$ (b) $3\vec{i}$

(c) $3\vec{i} - 4\vec{j}$ (d) $3\vec{i} - 6\vec{k}$

Q. 73 Let Ø be an arbitrary smooth real valued scalar function and V be an arbitrary smooth vector valued function in a three dimensional space. Which one of the following is an identity ?

(a) Curl($\emptyset\vec{V}$) = ∇($\emptyset$Div$\vec{V}$)

(b) Div$\vec{V}$ = 0

(c) Div Curl$\overrightarrow{V}$ = 0

(d) Div(($\emptyset\vec{V}$) = Ø Div $\vec{V}$

Q. 74 The magnitude of the directional derivative of the function f(x,y) = $x^2 + 3y^2$ in a direction normal to the circle $x^2 + y^2 = 2$, at the point (1,1) , is

(a) $4\sqrt{2}$

(b) $5\sqrt{2}$

(c) $7\sqrt{2}$

(d) $9\sqrt{2}$

Q. 75 Consider points P and Q in the xy – plane, with P = (1,0) and Q = (0,1). The line integral $2\int_P^Q (xdx + ydy)$ along the semicircle with the line segment PQ as its diameter is

(a) -1

(b) 0

(c) 1

(d) Depends on the direction(clockwise or anticlockwise) of the semicircle.

Q. 76 Value of the integral $\oint_C (xydy - y^2dx)$, where, c is the square cut from the first quadrant by the lines x = 1 and y = 1 will be (use Green's theorem)

(a) 1/2

(b) 1

(c) 3/2

(d) 5/3

Q. 77 The value of $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$, (where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1) is

(a) 1.666

(b) 4

(c) 0

(d) 20

Q. 78 Which one of the following describes the relationship among the three vectors, $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + 3\vec{j} + \vec{k}$ and $5\vec{i} + 6\vec{j} + 4\vec{k}$?

(a) The vectors are mutually perpendicular
(b) The vectors are linearly dependent
(c) The vectors are linearly independent
(d) The vectors are unit vectors

Q. 79 Curl of vector $\vec{F} = x^2z^2\vec{i} - 2xy^2z\vec{j} + 2y^2z^3\vec{k}$ is

(a) $(4yz^3 + 2xy^2)\vec{i} + 2x^2z\vec{j} - 2y^2z\vec{k}$
(b) $(4yz^3 + 2xy^2)\vec{i} - 2x^2z\vec{j} - 2y^2z\vec{k}$
(c) $2xz^2\vec{i} - 4xyz\vec{j} + 6x^2z\vec{k}$
(d) $2xz^2\vec{i} + 4xyz\vec{j} + 6y^2z\vec{k}$

Q. 80 The surface integral $\iint_S \frac{1}{\pi}(9x\vec{i} - 3y\vec{j}).\vec{n}dS$ over the sphere given by $x^2 + y^2 + z^2 = 9$ is

(a) 200
(b) 220
(c) 186
(d) 216

Q. 81 If the function f(x) = $\sqrt{x^2 - 4}$ in [2, 4] satisfies the Lagrenge's mean value theorem, then there exists some c ∈ (2, 4). What is the value of c

(a) 12
(b) 6
(c) $\sqrt{2}$
(d) $\sqrt{6}$

Q. 82 The derivative of the function f(x) = sinnx is

(a) Odd function
(b) Constant function
(c) Even function
(d) None of the above

Q. 83 For what value of k, the function f(x,y) = $\begin{cases} \frac{sin^{-1}(xy-2)}{tan^{-1}(3xy-6)}, & (x,y) \neq (1,2) \\ k & ,(x,y) = (1,2) \end{cases}$ is continuous?

(a) ½ (b) 1/3

(c) ¼ (d) 3/4

Q. 84 Let f(x,y) = $\begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, x^2 + y^2 \neq 0 \\ 0 \quad , x = y = 0 \end{cases}$

(a) f(x,y) is continuous at origin (b) f(x,y) is not differentiable at origin

(c) $f_x(0,0) = f(0,0)$ (d) $f_y(0,0) = f(0,0)$

Q. 85 If f(x,y) = $2x^2 - xy + 2y^2$, then at (1, 2)

(a) $\frac{\partial f}{\partial x} = 2$ (b) $\frac{\partial f}{\partial y} = 7$

(c) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ (d) $\frac{\partial f}{\partial x} \neq \frac{\partial f}{\partial y}$

Q. 86 Given the function f(x,y) = $x^2 - 2xy + y^2 + x^3 - y^3 + x^5$. Then the function has

(a) Maximum value at origin (b) Minimum value at origin

(c) Neither maximum nor minimum value at origin (d) Maximum value but no minimum value at origin

Q. 87 Given the function f(x,y) = $x^3 + y^3 - 63(x + y) + 12xy$

(a) The function has four stationary points (b) The function is maximum at (-7, -7)

(c) The function is minimum at (3, 3) (d) The function has neither minimum nor a maximum at (5, -1)

Q. 88 Let f(x,y) = $\begin{cases} x^2 + 2y, \ (x,y) \neq (1,2) \\ 0 \quad , \ (x,y) = (1,2) \end{cases}$ then

(a) f(x,y) is continuous at (1, 2) (b) f(x,y) is discontinuous at (1, 2)

(c) f(x,y) has removable discontinuity at (1, 2) (d) $\lim_{(x,y)\to(1,2)} f(x,y)$ does not exist

Q. 89 If f(x,y) = $\begin{cases} xytan\left(\frac{y}{x}\right), (x,y) \neq (0,0) \\ 0 \quad , \ (x,y) = (0,0) \end{cases}$. Then at (0, 0)

(a) $xf_x + yf_y = 2f$

(b) $xf_x - yf_y = 2f$

(c) $yf_x + xf_y = 2f$

(d) $yf_x - xf_y = 2f$

Q. 90 If f(x,y) = x^y , then

(a) f_x (a, 0) = 1, where a is a constant

(b) f_y (e, 0) = 1

(c) f_{xy} (1, 0) = 1

(d) f_{yx} (1, 1) = 1

Q. 91 Let the function f be defined as follows f(x) = $\begin{cases} \frac{1}{2^n}, \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}, n = 0,1,2 \dots \\ 0 \quad , else \end{cases}$ Then $\int_0^1 f(x)dx$ is

(a) 1

(b) 0

(c) 2/3

(d) 3/2

Q. 92 The value of integral $\int_0^\infty e^{-x^2} cos\alpha \ x \ dx$ is

(a) $\frac{1}{2}\sqrt{\pi}\, e^{-\frac{1}{4}\alpha^2}$

(b) $\frac{1}{2}\sqrt{\pi}$

(c) $\sqrt{\pi/6}$

(d) $\frac{1}{2}\alpha^2$

Q. 93 If f is an increasing function, then

(a) –f is decreasing function

(b) –f is increasing function

(c) –f is constant

(d) None of the above

Q. 94 The function f(x) = $\begin{cases} \frac{sinx}{x}, x \neq 0 \\ 0 \;, \; x = 0 \end{cases}$ has

(a) Discontinuity of first kind at x = 0
(b) Discontinuity of second kind at x = 0
(c) Continuity at x = 0
(d) None of the above

Q. 95 The function sinx^n is

(a) differentiable
(b) non-differentiable
(c) discontinuous
(d) None of the above

Q. 96 If x = a(t + sin t), y = a(1 – cos t) then dy/dx at t = $\pi/2$ is

(a) 2
(b) 1
(c) 5
(d) 8

Q. 97 Let y = x^x , then dy/dx is equal to

(a) x^x (1 + log x)
(b) 1 + log x
(c) log x
(d) $\frac{logx}{x}$

Q. 98 A point on a curve is said to be an extemum if it is a local minimum or a local maximum. The number of distinct exterma for the curve $3x^4 – 16x^3 -24x^2 +37$ is

(a) 0
(b) 1
(c) 2
(d) 3

Q. 99 Let f(x) = xe^{-x} . The maximum value of the function in the interval (0, ∞) is

(a) e^{-1}
(b) e
(c) 1 - e^{-1}
(d) 1 + e^{-1}

Q. 100 The length of the curve y = $\frac{2}{3}x^{\frac{3}{2}}$ between x = 0 and x = 1 is

(a) 0.27 (b) 0.67

(c) 1 (d) 1.22