$DRONACHARYA\ COLLEGE\ OF\ ENGINEERING, GURGAON$

$Department\ of\ Applied\ Sciences\ and\ Humanities$

$Subject: Mathematics – I\ (MATH - 101 - F)$

Question Bank

Short Type Question

Q.1 Discuss the convergence & divergence of the geometric series.

Q.2 $\sum_{n=1}^{\infty}(\sqrt[3]{n^3+1}-n)$

Q.3 $\sum(\sqrt{n^4+1}-\sqrt{n^4-1})$

Q.4 $\sum\frac{\sqrt{n}}{n^2+1}$

Q.5 $\sum\{\sqrt[3]{n+1}-\sqrt[3]{n}\}$

Q.6 Test the converegence of the series whose nth term is $\frac{n^{n^2}}{(n+1)^{n^2}}$

Q.7 Give the statement of D'Alembert ratio test & Gauss test.

Q.8. Test the convergence of the series $\sum\frac{2n^3+5}{4n^5+1}$

Q.9 Test the convergence of the series $\sum\frac{n^2}{2^n}$

Q. 10 Test the convergence of the series $\sum\frac{1}{n}\sin\frac{1}{n}$

Q.11 Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$

Q.12 Test the convergence of the series $\sum(\sqrt{n^4+1}-\sqrt{n^4-1})$

Q.13 Test the convergence of the series $\sum\frac{n^2(n+1)^2}{n!}$

Q.14 Test the series for convergence and absolute convergence:

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ \dots\dots\dots\dots\dots\dots\dots.\infty$$

Q.15 Discuss the convergence of the series:

$$1-\frac{1}{3}+\frac{1}{2}-\frac{1}{3^3}+\frac{1}{2^2}-\frac{1}{3^5}\dots\dots\dots\dots\dots\dots\dots.\infty$$

SECTION -II

Q.1 Define elementary matrices with exmple

Q.2 The rank of the matrix $\begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$ is............

Q.3 Define rank of a matrix & find The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is...

Q.4 The inverse of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ is......................

Q.5 Is the matrix $\begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ orthogonal ?

Q.6 The eigen value of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ are...................

Q.7 The sum of the eigen values of $\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 6 & 4 & 6 \\ 7 & 3 & 8 & 2 \\ 4 & 3 & 0 & 5 \end{bmatrix}$ is............

Q.8: The product of the eigen values of $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ is...................

Q.9 The eigen values of the matrix $\begin{bmatrix} 2 & 7 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & 5 \end{bmatrix}$ are.....................

Q.10 The characteristic equation of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ is.........

Q.11 Using Cayley Hamilton theorem find the inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

Q.12 Are the vectors (1,2,1),(2,1,4) & (4,5,6) are linear dependent ?

Q.13 What will be the product of eigen values of singular Matrix

Q.14 using Cayley-hamilton theorem , find $A^6\ if\ A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

Q.15 Determine the nature of the Quadratic form $2x^2 + 2y^2 + 3z^2 + 2xy - 4yz - 4xz$

SECTION -III

Q.1. If u $= \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

Q. 2 Prove that $Expand\ tanx\ in\ powers\ of\ \left(x - \frac{\pi}{4}\right)\ by\ using\ Taylor'sseries.$

Q.3 Prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ \ldots\ldots$

Q.4 If u $= \frac{x^3y^3}{x^3+y^3}$, show that $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 3V$

Q.5 If $= sin^{-1}\frac{x^2+y^2}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu$

Q.6 If u $= sin^{-1}\frac{x}{y} + tan^{-1}\frac{y}{x}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Q.7 If $u = e^{xyz}$ then find $\frac{\partial^3 u}{\partial x \partial y \partial z}$

Q.8 If $x = r\sin\theta\ \cos\emptyset$, $y = r\sin\theta\ \sin\emptyset$, $z = rcos\theta, then\ find\ \frac{\partial(r,\theta,\emptyset),}{\partial(x,y,z)}$

Q.9 Find the asymptotes of the curve $x^3 + y^3 - 3axy = 0$

Q.10 Find the asymptotes parallel to co-ordinates of the curve

$$x^2y^2 - x^2y - y^2x + x + y + 1 = 0$$

Q.11 Find the asymptotes of the following curves $r = a\tan\theta$

Q.12 Expand a^x by using Maclaurine's series

Q.13 If u = $x^3 + y^3$, where x = a cos t , y = b sin t , find du / dt

Q.14 Examine the extreme values of $x^2 + y^2 + 6x + 12$

Q.15 State Leibnitz's Rule (Differentiation under integral sign).

Section-IV

Q.1 Evaluate $\int_0^1 \int_1^{\sqrt{1-x^2}} (y^2)dxdy$ by changing the order of integration.

Q.2 Change the order of integration $\int_0^a \int_y^a \left(\frac{x}{x^2+y^2}\right) dxdy$

Q.3 Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dzdydx$

Q.4 write the formula of finding area & volume for double integration.

Q.5 Evaluate $\iint_R xydxdy$, where R is the first quadrant of the circle $x^2 + y^2 = a^2$, where x,y$\geq$ 0

Q.6 Evaluate $\iint xy(x^2 + y^2)$dxdy over the positive quadrant of the circle $x^2+y^2 = 1$

Q.7 Write the formula of finding the volume of the solid generated by revolving about both the coordinates axis.

Q.8 Evaluate the integral $\int_0^{\pi} \int_0^{a\, sin\theta} r^2\, dr\, d\theta$

Q.9 Transform the integral to Cartesian form $\int_0^{\pi} \int_0^{a} \mathrm{r}^3 \sin\theta \cos\theta \,\mathrm{dr}\, \mathrm{d}\theta$

Q.10 Prove that $\int_0^1 x^4 \left(log\frac{1}{x}\right)^3 dx = \frac{6}{625}$

Long Type Question

SECTION -I

Test the convergence of the following series.

Q1. $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \left(\frac{1.2.3.4}{3.5.7.9}\right)^2 + \dots\dots\dots\dots\dots..\infty$

Q2. $\sum \frac{n}{n^2+1} x^n$, x > 0

Q3. $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots\dots\dots\dots\dots\infty$

Q4. $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots\dots\dots\dots\dots\dots\dots\dots....\infty$

Q5. $\sum \left(\frac{nx}{n+1}\right)^n$

Q6. $\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \infty$

Q7. $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. \infty$

Q8. $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \frac{5^5x^5}{5!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \infty$

Q9 $1 + \frac{1}{2} . \frac{x^2}{4} + \frac{1.3.5}{2.4.6} . \frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10} . \frac{x^6}{12} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \infty$

Q10. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(logn)^p}$, $(p > 0)$

Q11. Test the convergence of the series

$$\sum \left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \left(\frac{1.2.3.4}{3.5.7.9}\right)^2 + ------\infty$$

Q.12. Test the convergence of the series $\sum \frac{x^n}{(2n)!}$

Q13. Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + ---(x > 0)$$

Q14. Test the convergence of the series

$$1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)}x^2 + \frac{a(a+1)(a+2)}{b(b+1)(b+2)}x^3 + ------\infty$$

Q15. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

$p > 1$ *and diverges if* $0 < p \leq 1$

Q16. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(logn)^p}$, $(p > 0)$

Q17. Examine the convergence of the series

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + --$$

Q18. Discuss the convergence of the series

$$\frac{x}{x+1} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + ------- \infty$$

Q19. Test the convergence and absolute convergence of the series

$$\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{5}+1} + ------- \infty$$

Q20. Test the convergence and absolute convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}.n}{n^2+1}$$

Q21. Prove that the series converges absolutely

$\sum_{n=1}^{\infty}(-1)^{n-1}.\frac{n^3}{(n+1)!}$

Q22. Show that the series $\frac{log2}{2^2} - \frac{log3}{3^2} + \frac{log4}{4^2} - \;\ldots\ldots\ldots \infty$ converges.

Q23. Prove that the series $\frac{sinx}{1^3} - \frac{sin2x}{2^3} + \frac{sin3x}{3^3} - \;\ldots\ldots$ converges absolutely.

Q24. Discuss the convergence and absolute convergence of series

$x - \frac{x^3}{3} + \frac{x^5}{5} - \;\ldots\ldots\ldots\ldots\ldots\ldots..\infty$, x being real.

Q25. For what values of x, the series converges:

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \;\ldots\ldots\ldots\ldots\ldots..\infty$

SECTION -II

Q1 Use Gauss-Jordan method, find the inverse of the matrix

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Q2 Determine the rank of the matrix by reducing it to the normal form

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

Q3 Find non-singular matrices P and Q such that PAQ is in the normal form for the matrix

$$\begin{bmatrix} 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Q4 Test for consistency and solve the equations

$x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$, $3x + 9y - z = 4$

Q5 For what values of a and b do the equations $x + 2y + 3z = 6$,

$x + 3y + 5z = 9$, $2x + 5y + az = b$ have (i) unique solution

(ii) no solution (iii) more than one solution

Q6 Determine the values of λ for which the system may possess non-trivial solution $3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$.For each permissible value of λ, determine the general solution.

Q7 Are the vectors linearly dependent? If so, find a relation between them. $x_1 = (2,3,1,-1)$, $x_2 = (2,3,1,-2)$, $x_3 = (4,6,2,1)$

Q8 Represent each of the transformations $x_1 = 3y_1 + 2y_2$, $x_2 = -y_1 + 4y_2$, $y_1 = z_1 + 2z_2$, $y_2 = 3z_1$ by the use of matrices and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2.

Q9(i) Verify the matrix is orthogonal $\frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

(ii) If A and B are orthogonal matrices, prove that AB is also orthogonal .

Q10 Find the eigen values and eigen vectors of the matrices

(i) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Q11 State and prove the properties of eigen values.

Q12 Find the characteristic equation of the matrix

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$ Show that the equation is satisfied by A. Also find A^{-1}.

Q13 Using Cayley-Hamilton theorem, find A^6 ,if

$A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

Q14 Reduce the matrix into normal form and find its rank

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

Q15 Find the characteristic equation of the matrix

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ And, hence, find the matrix represented by

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

Q16 Diagonalise the matrices

(i) $\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$

Q17 Diagonalise $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^8.

Q18 Reduce the quadratic form into the canonical form and hence specify the matrix of transformation.

$3x^2 + 3y^2 + 3z^2 + 2xy + 2yz - 2zx$

Q19 Reduce the given quadratic form into the canonical form and hence specify the matrix of transformation $f(x,y)=8x^2 + 7y^2 + 3z^2 - 12xy + 4zx$

SECTION -III

Q1. If $= tan^{-1}\frac{xy}{\sqrt{1+x^2+y^2}}$, show that $\frac{\partial^2 u}{\partial x \partial y} = (1 + x^2 + y^2)^{-\frac{3}{2}}$

Q2. If $r^2 = x^2 + y^2 + z^2 and\ V = r^m, show\ that$

$V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-2}$

Q3. If x = r cosθ, y = r sinθ, prove that

$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$

Q4. If u = f(r) , where $r^2 = x^2 + y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

Q5. If $= tan^{-1} \frac{x^3+y^3}{x-y}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = sin4u - sin2u = 2cos3usinu$$

Q6. If $= cosec^{-1} \left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{tanu}{144}(13 + tan^2 u)$$

Q7. At a given instant the sides of a rectangle are 4 ft. and 3 ft. respectively and they are increasing at the rate of 1.5 ft/sec. and 0.5 ft./sec. respectively. Find the rate at which the area is increasing at that instant.

Q8. If u = f(r, s, t) and $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$,

$$prove\ that\ x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

Q9. If $u = \frac{y^2}{2x}$, $v = \frac{x^2+y^2}{2x}$, $find\ \frac{\partial(u,v)}{\partial(x,y)}$

Q10. If $u = \frac{x+y}{1-xy}$ $and\ v = tan^{-1}x + tan^{-1}y$, $find\ \frac{\partial(u,v)}{\partial(x,y)}$

Q11. If u = x + y + z, uv = y + z, uvw = z,

show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$

Q12. Expand e^{ax} sinby in powers of x and y as far as the terms of

third degree.

Q13. Expand sin xy in powers of (x – 1) and (y – π/2) upto the second

degree terms.

Q14. Examine the extreme values of $x^3 + y^3 - 63(x + y) + 12xy$

Q15. Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

Q16. A rectangular box open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

Q17. Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

Q18. Evaluate the integral $\int_0^{\infty} e^{-x} \frac{sinax}{x} dx$

Q19. Prove that $\int_0^{\infty} e^{-x} \frac{1-e^{-ax}}{x} dx = \log(1+a), \quad (a \geq 0)$

Q20. Using $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}, show\ that$

$$\int_0^{\infty} e^{-x^2} cos\alpha x\ dx = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{4}\alpha^2}$$

Q21. Prove that $\int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} sin\theta cos^{-1}(cos\alpha\ cos\theta) d\theta = \frac{\pi}{2}(1 - cos\alpha)$

Q22. Evaluate $\int_0^{a} \frac{\log(1+ax)}{1+x^2}\ dx$ and hence show that $\int_0^{1} \frac{\log(1+x)}{1+x^2}\ dx = \frac{\pi}{8} log2$

SECTION -IV

Q.1 Evaluate $\int_0^{2a} \int_0^{\sqrt{ax-x^2}} (x^2 + y^2) dydx$

Q.2 Evaluate $\iint_A\ xydxdy$, where A is the domain bounded by x – axis, ordinate x = 2a and the curve $x^2 = 4ay$.

Q.3 Show that $\iint_R\ r^2 sin\theta\ drd\theta = \frac{2a^3}{3}$, where R is the region bounded by the semi circle r = 2acosθ, above the initial line.

Q.4 Evaluate $\iint r^3 drd\theta$ over the area included between the circles r = 2a cosθ, r = 2b cosθ, (b < a).

Q.5 Evaluate the integral by changing the order of integration

$$\int_0^{a} \int_{\frac{x^2}{a}}^{2a-x} xy\ dydx.$$

Q.6 Evaluate the integral by changing the order of integration $\int_0^{\infty} \int_0^{x} x e^{-\frac{x^2}{y}}\ dydx.$

Q.7 Evaluate the triple integral $\int_0^{log2} \int_0^{x} \int_0^{x+logy} e^{x+y+z}\, dzdydx$.

Q.8 Evaluate the integral by changing into polar co-ordinates:

$\int_0^a \int_y^a \frac{x\, dxdy}{x^2+y^2}$

Q.9 Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$, over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Q.10 Show, by double integration, that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

Q.11 Find, by double integration, the volume of the sphere $x^2 + y^2 + z^2 = 9$.

Q.12 Prove that the volume enclosed between the cylinder $x^2 + y^2 = 2ax$ and $z^2 = 2ax$ is $\frac{128}{15}a^3$.

Q.13 Find, by triple integration, the volume in the positive octant bounded by the coordinate planes and the plane $x + 2y + 3z = 4$.

Q.14 Find, by double integration, the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis.

Q.15 State and prove the relation between Beta and Gamma functions.

Q.16 Prove that $\int_0^1 x^4 \left(log\frac{1}{x}\right)^3 dx = \frac{6}{625}$

Q.17 Prove that $B\left(m,\frac{1}{2}\right) = 2^{2m-1}B(m,m)$

Q.18 Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

Q.19 Prove that $\int_0^1 \left(log\frac{1}{y}\right)^{n-1} dy = \Gamma(n),\ \ n > 0$

Q.20 Show that $B(p,q) = \int_0^{\infty} \frac{y^{q-1}}{(1+y)^{p+q}}dy$.