Boolean Algebra

BOOLEAN ALGEBRA

- Boolean algebra provides the operations and the rules for working with the set **{0, 1}.**

- These are the rules that underlie **electronic circuits**, and the methods we will discuss are fundamental to **VLSI design**.

- We are going to focus on three operations:
  - Boolean complementation,
  - Boolean sum, and
  - Boolean product

BOOLEAN OPERATIONS

- The **complement** is denoted by a bar (on the slides, we will use a minus sign). It is defined by

$\overline{0} = 1$ and $\overline{1} = 0$.

- The **Boolean sum**, denoted by + or by OR, has the following values:
- $1 + 1 = 1, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 0 + 0 = 0$
- The **Boolean product**, denoted by · or by AND, has the following values:
- $1 \cdot 1 = 1, \quad 1 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 0 \cdot 0 = 0$

BOOLEAN FUNCTIONS AND EXPRESSIONS

- There is a simple method for deriving a Boolean expression for a function that is defined by a table. This method is based on **minterms**.
- **Definition:** A **literal** is a Boolean variable or its complement. A **minterm** of the Boolean variables x_1, x_2, …, x_n is a Boolean product $y_1 y_2 \ldots y_n$, where $y_i = x_i$ or $y_i = -x_i$.
- Hence, a minterm is a product of n literals, with one literal for each variable.

BOOLEAN FUNCTIONS AND EXPRESSIONS

- **Definition:** The Boolean functions F and G of n variables are **equal** if and only if $F(b_1, b_2, \ldots, b_n) = G(b_1, b_2, \ldots, b_n)$ whenever $b_1, b_2, \ldots, b_n$ belong to B.
- Two different Boolean expressions that represent the same function are called **equivalent**.
- For example, the Boolean expressions xy, $xy + 0$, and $xy \cdot 1$ are equivalent.

BOOLEAN FUNCTIONS AND EXPRESSIONS

- **Question:** How many different Boolean functions of degree 1 are there?
- **Solution:** There are four of them, F_1, F_2, F_3, and F_4:

x	F_1	F_2	F_3	F_4
0	0	0	1	1
1	0	1	0	1

BOOLEAN FUNCTIONS AND EXPRESSIONS

- **Question:** How many different Boolean functions of degree 2 are there?
- **Solution:** There are 16 of them, F_1, F_2, ..., F_{16}:

x	y	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

BOOLEAN FUNCTIONS AND EXPRESSIONS

- **Question:** How many different Boolean functions of degree n are there?
- **Solution:**
- There are 2^n different n-tuples of 0s and 1s.
- A Boolean function is an assignment of 0 or 1 to each of these 2^n different n-tuples.
- Therefore, there are 2^{2^n} different Boolean functions.

DUALITY

- There are useful identities of Boolean expressions that can help us to transform an expression A into an equivalent expression B

- We can derive additional identities with the help of the **dual** of a Boolean expression.

- The dual of a Boolean expression is obtained by interchanging Boolean sums and Boolean products and interchanging 0s and 1s.

DUALITY

- **Examples:**

The dual of $x(y + z)$ is $x + yz$.

The dual of $\overline{x}\cdot 1 + (\overline{y} + z)$ is $(\overline{x} + 0)((\overline{y})z)$.

The dual of a Boolean function F represented by a Boolean expression is the function represented by the dual of this expression.

This dual function, denoted by F^d, does not depend on the particular Boolean expression used to represent F.

DUALITY

- Therefore, an identity between functions represented by Boolean expressions **remains valid** when the duals of both sides of the identity are taken.

- We can use this fact, called the **duality principle**, to derive new identities.

DEFINITION OF A BOOLEAN ALGEBRA

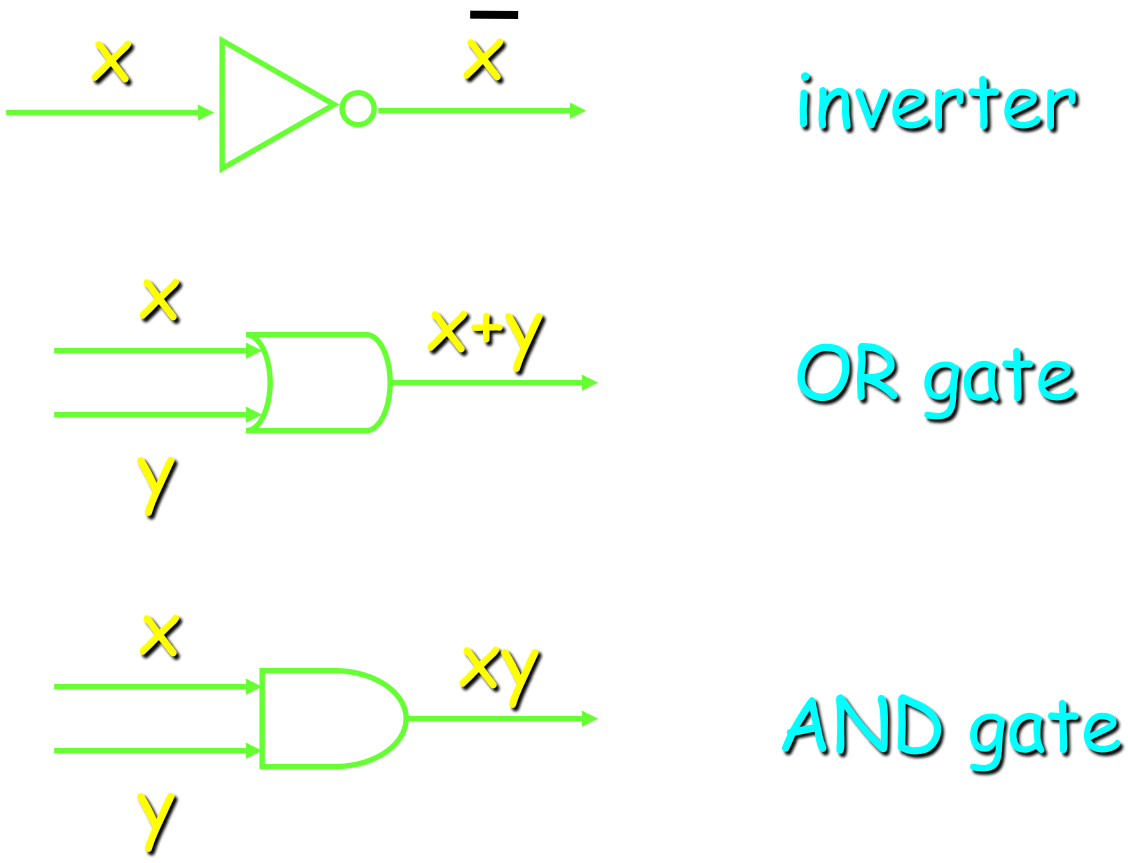
- **All the properties of Boolean functions and expressions that we have discovered also apply to other mathematical structures such as propositions and sets and the operations defined on them.**

- **If we can show that a particular structure is a Boolean algebra, then we know that all results established about Boolean algebras apply to this structure.**

- **For this purpose, we need an abstract definition of a Boolean algebra.**

LOGIC GATES

- Electronic circuits consist of so-called gates. There are three basic types of gates:



LOGIC GATES

- **Example:** How can we build a circuit that computes the function $xy + (\overline{x})y$?